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Managing Marginal Capital Requirements in Solvency II: Analysis of a Real Insurance Portfolio

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Abstract

This thesis provides a complete analysis of the Standard Capital Requirements given by Solvency II for a real insurance portfolio. We analyze the investment portfolio of BPI Vida e Pensões, an insurance company affiliated with a Portuguese bank BPI, both at security, sub-portfolio and asset class levels. By using the Standard Formula from EIOPA, Total SCR amounts to 239M€. This value is mostly explained by Market and Default Risk whereas the former is driven by Spread and Concentration Risks.

Following the methodology of Leblanc (2011), we examine the Marginal Contribution of an asset to the SCR which allows for the evaluation of the risks of each security given its characteristics and interactions in the portfolio. The top contributors to the SCR are Corporate Bonds and Term Deposits. By exploring further the composition of the portfolio, our results show that slight changes in allocation of Term and Cash Deposits have severe impacts on the total Concentration and Default Risks, respectively. Also, diversification effects are very relevant by representing savings of 122M€.

Finally, Solvency II represents an opportunity for the portfolio optimization. By constructing efficient frontiers, we find that as the target expected return increases, a shift from Term Deposits/ Commercial Papers to Eurozone/Peripheral and finally Equities occurs.

Contents

1 Introduction

The acknowledgement of the risks that a company faces is crucial for all firms in the financial framework, especially insurance companies as their business strongly depends on both business liquidity and ability to fulfill their client's demands. The necessity for a transparent, comparable and complete assessment of the risk exposure, very important for the every day activities of an insurance institution, motivated the European Insurance and Occupational Pensions Authority (EIOPA) to launch the Solvency project.

The capital charges for an European insurance company started with Solvency I, which imposed capital requirements (i.e., the capital needed to face the risks of the insurance company) as 1% of the value of its total assets if the portfolio had no capital guarantee towards the client, and 4% if these guarantees existed. However, this approach was insufficient given that it assumed that the composition of the insurance portfolio was irrelevant and risk was dependent merely on the value of assets and on the existence of capital guarantees. Thus, the Solvency II project, initiated in 2000, considers both market-consistent valuations and asset-side risk, resulting in a risk-sensitive measure: the Standard Capital Requirement (SCR).

This thesis provides the most complete analysis of the Standard Capital Requirement for a real insurance portfolio. Using the insurance portfolio of BPI Vida e Pensões, an insurance company affiliated with a Portuguese commercial bank BPI, we compute the total SCR and test preliminary hypothesis based on the composition of the portfolio. Total SCR is dominated by both Market and Default Risk, in which the former is driven mostly by Spread and Concentration Risk. Moreover, this thesis examines the marginal contribution of an asset for the Standard Capital Requirements under Solvency II, following the methodology of Leblanc (2011). To the best of my knowledge, we are the first to apply this methodology to a real portfolio, enabling to understand what are the securities that contribute most to the SCR and thus, to provide real information to a portfolio manager. The analysis starts at a portfolio level using the Standard Model developed by EIOPA and focus on the Market and Default Risk. Then, the overall

results are broken down into asset classes, individual securities and sub-portfolios, in order to evaluate how the capital requirements can be explained under these perspectives. In fact, we find that Corporate Bonds and Term Deposits lead the rankings in terms of capital charges, with a high exposure to Spread and Concentration Risk, respectively and that this trend is observed also when analyzing individual securities. Moreover, the sub-portfolios with the highest market values are responsible for the largest consumption of capital but, at the same time, are the ones with the lowest SCR as a percentage of assets due to diversification gains. We explore the diversification gains of the company and find that it saves up 122M€ of capital and find that the total SCR is very sensitive to changes in the correlation matrix provided by EIOPA. Finally, this work shifts towards optimal capital allocation given the portfolio in hands; that is, it is explored whether the current level of capital requirements of the portfolio may be optimized by changing the allocation of the different asset classes taking into consideration the expected return of the portfolio. This exercise allows for the design of the portfolios' efficient frontier and shows that the allocation to different asset classes change significantly depending on the target expect return of the portfolio.

The rest of the thesis is structured as follows: Section 2 presents a review of the relevant literature. Section 3 provides an overview of the methodology of the SCR under the Standard Formula provided by EIOPA and how the marginal contribution is calculated. Section 4 presents summary statistics of the portfolio. Section 5 shows preliminary hypothesis while Section 6 the results for the SCR and marginal contributions. Section 7 explores the Optimal Asset Allocation and Section 8 concludes.

2 Literature Review

According to EIOPA specifications, Solvency II segregates risks in two levels, as depicted in **Appendix 1**. Firstly, risk is divided into Market, Health, Default, Life, Non-Life and Intangibles. At a second level, Market Risk is decomposed into Interest Rate, Spread, Equity, Property, Currency and Concentration modules; while Life includes

Mortality, Longevity, Disability/Morbidity, Lapse, Expenses, Revision and Catastrophe. Therefore, by calculating the Standard Capital Requirement in this way, a risk-sensitive measure is obtained which treats assets differently according to their inherent risks, but also considers the diversification effect present in portfolios which hold a wide variety of securities in different asset classes.

As a result, portfolio managers may see in the implementation of Solvency II not only a necessity but also an opportunity to reallocate their asset portfolio in order to optimize their capital positions. In fact, Solvency I could allow for the undertaking of excessive risk (given the fixed percentage of capital requirements), implying that the measurement under the new rules would bring a capital burden that would need to be quickly managed. On the other hand, the fixed capital requirement of Solvency I could overestimate the risk of the portfolio, thus, the true level of risk could be well below the one previously fixed. Either way, under Solvency II, by knowing how each class impacts the capital requirements, the portfolio manager has the opportunity to manage the portfolio in a such a way that optimizes its capital expenditure, given a certain expected return.

According to Fitch Ratings (2011), Solvency II is set to transform the way insurers allocate their investments, given that Market Risk and thus, Solvency II capital requirements are very sensitive to changes in asset allocation. The agency believes that the main impacts will be a shift from long-term to short-term debt, increasing attractiveness of bonds from corporations with higher rating and government bonds, namely from countries belonging to the EEA/OECD. They also bet on a greater necessity for diversification, preference for covered bonds and assets protected by swap rates and, lastly, a preference towards deposits instead of short-dated paper.

Finally, Fitch Ratings also supports the theory that changes in asset allocation will depend both on the capital requirements of an asset class or security and on their expected return, whereas the insurer will try to construct a portfolio with lower capital charges for a given level of return.

The relationship between Solvency II and optimal asset allocation is also subject of analysis by Jarraya and Bouri (2013). In their work, they identify an additional constraint in terms of asset management for a insurer, when compared to Solvency I. On the one hand, in the Solvency I framework the asset manager's focus would be on managing a portfolio with liability constraints, but also on the necessity to cover the investment risk made, which will be determined as a fixed percentage of the assets. On the other hand, under Solvency II, the capital requirements will depend on the portfolio composition, which by not being a fixed percentage of the total assets, will affect the management of the assets in the portfolio and thus, place an additional constraint.

Yet, it is important to understand that all the calculations of Solvency II provided by EIOPA for the Standard Model aim to provide an overall risk amount for the portfolio, starting at a individual level for each class of risk but aggregating its content to get the capital requirements per risk and, after applying the correspondent correlation matrices, the final capital requirements for the portfolio/company. However, the analysis of risk per security requires that this standard approach is analyzed further and decomposed in marginal effects per individual securities, which is the main goal of this work. For such, Leblanc (2011) has developed an initial mathematical framework for marginal decomposition, which will be the basis for the following computations and analysis and establishment of the optimal asset allocation portfolio. This previous work is now extended by applying the marginal methodology to a real portfolio of an insurance company and by allowing the Market Risk components to be decomposed, taking into consideration the correlations between the modules.

Most papers have focused on the capital charges of an individual asset class and its expected return, without taking into consideration its interaction with the base portfolio in which it is inserted. As we know, the correlations between asset classes severely influence the capital charges and thus, a complete analysis must be performed. Therefore, it is shown how changes in the asset allocation impact the capital requirements in the portfolio in analysis, not only due to the inherent risks of the security but also

due to its diversification benefits. A correlation sensitivity analysis is also performed in order to show how different parameters severely change the total SCR.

3 Methodology

BPI Vida e Pensões uses the Standard Model of Solvency II provided by EIOPA. Using this model, the company is able to reach the total Standard Capital Requirements set as the capital needed to face the risks of the insurance company and defined as the Value-at-Risk at 99,5% of the Basic Own Funds for a time horizon of one year, taking into account risk mitigation techniques and diversification effects. In this context, Basic Own Funds corresponds to the difference between asset and liabilities, despite of certain adjustments (subordinated liabilities are added back, treasury share are deducted and the recognition of reserves is limited and based on their availability to offset losses).

Regardless of the main goal of the work being the calculation of the marginal effect of different financial instruments in the SCR and total asset value, it is crucial to compute the SCR of the insurance company and, only then, calculate and analyze the marginal effect. The calculation of the SCR is based on a bottom-up approach, starting with the calculation of the capital requirement for each sub-module of risk, then aggregated with resort to a correlation matrix provided by EIOPA, leading to the overall value of each module. Having reached the individual module SCRs, a similar method is applied and the capital requirement for the insurance company is obtained by aggregating these modules together, once again taking into account their correlations. The resulting capital charge corresponds to the amount the insurance company needs to hold in order to face the risks incurred and is defined as the Basic Solvency Capital Requirement (BSCR). In order to obtain the final value for the Standard Capital Requirements, the SCR for Operational Risk is added (See description of this risk in **Appendix 2)**. Finally, an adjustment is made due to the future discretionary benefits and deferred taxes.

3.1 Standard Capital Requirement

The total Standard Capital Requirement as defined beforehand is obtained through the sum between the Basic Solvency Capital Requirements (BSCR), Operational SCR (SCR_{Op}) and the adjustment for the risk absorbing effect of technical provisions and deferred taxes (Adj):

$$SCR = BSCR + SCR_{Op} + Adj$$

The Basic Solvency Capital Requirements combines six major risk categories: Market Risk, Health Risk, Default Risk, Life Risk, Non-Life Risk and Intangibles. However, BPI Vida e Pensões operates solely with life insurance, implying that the company is only exposed to Market, Default and Life risks.

The adjustment for the risk absorbing effect of technical provisions and deferred taxes, assumed as an input for calculation purposes, includes the future discretionary benefits and the deferred taxes. These future discretionary benefits appear in opposition to mandatory benefits, are based in future performance (return or P&L) and can be contractually guaranteed or not.

The next section provides an explanation on the risks that influence the asset-side of the insurer, namely Market and Default Risk. Further details regarding Operational and Life Underwriting Risk, which are taken as an input and outside the scope of this work, can be found in **Additional Appendix 1 and 2.** The correlation matrix that combines all this risks can be found in **Appendix 2**. For all the risks considered, the choice between the several scenarios presented and used to compute the capital charges will be based on the value of the capital requirements including the loss absorbing capacity of technical provisions (or future discretionary benefits), being that the such is only used for scenario comparison effects and the final capital requirement for each risk module will not include such adjustments.

3.1.1 Market Risk

The Market Risk module covers the risk arising from fluctuations in market prices of securities. The main risk factors are Equity, Interest Rate, Property, Spread, Concentration and Currency Risk. The portfolio will be subjected to every risk except Property Risk which explanation can be found in **Additional Appendix 3**. For each one of these risks different shocks (up and down) will be applied, whereas the shock that constitutes the greatest capital requirement value will be chosen.

The SCR for the Market Risk is computed by aggregating the sub-modules SCR as it takes into account the correlation terms, according to the following formula:

$$SCR_{market} = \sqrt{\sum Corr_{i,j} \times SCR_i \times SCR_j}$$

where$i$ and j correspond to the different risk factors and $\sum Corr_{i,j}$ to the correlation between both risks (the Market correlation matrix can be found in **Appendix 3**).

Moreover, the look-through approach is followed: the securities that constitute the funds included in the portfolios of BPI Vida e Pensões will be discriminated and added into each of the respective sub-modules, in order to accurately assess the market risk taken by the insurance company.

Equity Risk

A portfolio containing Equities is exposed to their volatility and level of the market prices, which has a direct impact on the value of assets. Thus, Solvency II requires that an insurance company holds sufficient capital to absorb an atypical shock in the markets in order to avoid any major loss to the value of the portfolio. The universe of securities includes equities, hybrid securities (securities with both fixed income and equity characteristics) and also hedging and risk transfer mechanisms; while ignoring short positions. Moreover, investments made in companies engaging in real estate management are accounted as equity investments under alternative investments (such as the fund "Imofomento" in our universe of analysis).

The securities are divided considering the country of domicile and the type of investment, which will consequently affect the type of shocks applied. Type 1 equities are equities listed in regulated markets in countries belonging to the EEA and OECD or issued by authorized alternative investment funds, while Type 2 equities include all the remaining categories (non-EEA or OECD members, unlisted, hedge funds, commodities and other alternative investments). Additionally, EIOPA also distinguishes strategic participations from other equity investments, where a participation consists in a share ownership or exertion of a strong influence over the undertaking over a relatively long period of time.

Therefore, the SCR for equity risk corresponds to a percentage shock which reflects the capital required to accommodate a fall in the capital markets. Type 1 equities suffer a base downward shock of 39% while Type 2 equities face a 49% stress level. In addition to these base levels, it is also applied a symmetric adjustment which is based on the index levels for the markets considered and which, based on 2013 financials, implied an adjustment around 5%. Lastly, a shock of 22% is used for strategic participations, regardless of being Type 1 or 2 instruments. Given the non-perfect correlations between the two types, a 75% correlation factor is applied. Consequently, the market value for the capital requirements for equity risk is given by

$$SCR_{market} = \sqrt{\sum CorrIndex^{rxc} \times Mkt_r \times Mkt_c}$$

where $Mkt_r$and Mkt_c are the capital requirements for equity per individual category.

Interest Rate Risk

The changes in the level or volatility of interest rates should be accounted in portfolios with assets sensitive to these rates. Changes in overall levels of interest rates are divided into two main sources of variability: fluctuations in the basic risk-free and spread rates. Under Solvency II, the two sources are distinguished into the interest rate risk and spread risk sub-modules, respectively.

The Basic Risk-Free term structure is derived from the basic interest swap rate

(IRS) term structure for each maturity and currency, assuming the markets are liquid and transparent. These rates must be adjusted for credit risk due to the risk premium reflecting the bank's credit quality in the floating leg of a swap contract, which should not be included in the risk-free rate. Following CEIOPS' specifications, the following adjustment was made: first, the difference between the EURIBOR (Euro Interbank Offered Rate) and the Overnight Indexed Swap curve for equal maturities, for a period of one year, was computed; second, it is considered a 50 percent of the average of those differences with a cap and a floor of 35 and 10 basis points, respectively. The calculation of this adjustment is based on the Expectation Hypothesis, where, under no arbitrage opportunities, the rates paid on term bank deposits are linked to the expected overnight rates over the same period of time, as they tend to be interpreted as substitutes.

After obtaining the IRS curve adjusted for credit risk, it is required that rates are interpolated between data points (using a linear interpolation method) and extrapolated for longer term maturities (using the Smith-Wilson method, see **Additional Appendix 4** for more details). Having reached the adequate interest rate term structure, both the upward and downward shocks are applied and used to discount the future cash flows of fixed income related instruments. The magnitude of these shocks to yields varies, diminishing as maturity increases **(Additional Appendix 5).** Following the discounting of the securities' cash flows, it is calculated the difference between the regular scenario and both shocks. The capital requirement for interest rate risk corresponds to the shock that leads to the most unfavorable deviation from the original conditions.

However, and unlike other risks which affect only the asset side, this risk also affects the liabilities side of the insurance company, accounted for in the same way as assets but with opposite sign as changes in interest rates have different implications for assets and liabilities. Consequently, the overall Interest Rate Risk sub-module will introduce a new concept of Net Asset Value, which is defined as the capital requirements from assets deducted of the capital requirements from the liabilities side, that can, and will be, negative in this case (NAV concept is also present in other risks but as the "capital

requirements" from liabilities are 0, net asset value corresponds to the asset value).

Spread Risk

Spread Risk takes into consideration the risk of a change in the value of financial instruments due to deviations on credit spreads, that is, changes of the actual market price of credit risk from the expected price of credit risk. It includes covered bonds, corporate bonds, credit derivatives other than for hedging purposes, hybrid securities with fixed income characteristics, subordinated debt investments, loans and securitization positions. In fact, and given the non-existence of credit derivatives in the portfolio, the total Spread risk will be the sum of the Market Spread Risk for Bonds and Securitisations, that is:

$$Mkt_{sp} = Mkt_{sp}^{bonds} + Mkt_{sp}^{securitisation}$$

For the calculation of Spread risk's SCR, Mkt_{sp}, three inputs are needed: the market value of the credit risk exposure i (MV_i), the external credit quality step of an exposure i ($rating_i$), and the duration of the credit risk exposure i ($duration_i$). Given these characteristics, a fixed and a variable factor (F^{up}) will be set depending on both the credit quality step of the exposure and its modified duration, following reference tables provided by EIOPA (See **Additional Appendix 6**). Consequently, this factor is applied to the market value of the security, applying the shock in both bonds and securitisation positions. The credit quality step (CQS) in EIOPA specificities corresponds to an internal measure of credit quality values from 1 to 6 and which relate to the ratings obtained by Fitch, Moody's and Standard and Poor's. A conversion table between agency ratings and CQS is available and is presented in **Additional Appendix 7.**

Moreover, according to EIOPA specifications, other assumptions are considered: in case where several credit quality steps are available, the second-best is taken into consideration; covered bonds have a special treatment, following a different reference table for the risk factors as **Additional Appendix 6** also shows; the risk factor of 0% applies for government and central banks' bonds denominated in any currency from the

European Union.

Concluding, the SCR for Spread Risk constitutes the change in the Basic Own Funds required to withhold these shocks and is expressed in the following formula:

$$SCR_{sp} = \sum MV_i \times F^{up}(rating_i; duration_i)$$

Currency Risk

Currency Risk arises when there are changes in the level or volatility of the exchange rates different from the base currency. Securities expressed in currencies other than Euro will be subjected to appreciation/depreciation accordingly to the shocks in the financial markets. Therefore, and especially in the case of Euro depreciation against another currency, the insurance company needs to be prepared to absorb the resulting losses. The capital requirement for currency risk is obtained by applying both an upward and a downward shock, both of 25% and by taking the maximum between the capital requirements given by the shock up and down (with a floor of 0).

Concentration Risk

Concentration Risk is the risk associated with the probability of loss due to an accumulation of exposures of financial instruments to the same counterparty. It takes into consideration assets in the equity, spread and property sub-modules, but excludes assets covered in the counterparty default risk (for instance, cash deposits) in order to avoid overestimation of the SCR due to overlap between the two modules. Moreover, this module uses the look-through approach in the case of investment funds, thus, taking into consideration not the exposure to the fund itself, but the exposure to each sub-counterparty included in it.

For the calculation of the Concentration Risk SCR (Mkt_{conc}), the assets are grouped according to the counterparts involved, thus, deriving the *Net Exposure at Default* (E_i) to Counterparty i while taking into consideration the External Credit Quality Step of the same Counterparty i (*rating* i) and the total amount of assets considered in this

sub-module of risk ($Assets_{xl}$). In particular, the rating i of counterparty is defined as the weighted average of the individual credit quality steps assigned to that counterparty, weighted by the net exposure at default of individual securities to that company. Then, three steps are performed: first, the relative excess exposure to a counterparty (XS_i) is calculated, i.e., each counterparty will have an allowed threshold of exposure given its credit quality step (CT) (See **Additional Appendix 8**) and the excess exposure relative to that same threshold is computed:

$$XS_i = max(0, \frac{E_i}{Assets_{xl}} - CT)$$

Second, the risk concentration capital requirement per each exposure is computed as the expected loss in the Basic Own Funds in the case of a negative shock in the value of the assets of a given counterparty i. The shock depends on both the credit quality of the counterparty and the relative excess exposure previously computed. Specifically, the following formula determines the shock:

$$XS_i \times g_i,$$

where the parameter g_i depends on the credit quality step of the counterparty (See **Additional Appendix 8**). In particular, a risk factor of 0% applies to exposures to Member States of the European Union's central banks and governments denominated in any domestic currency of a EU Member state, to exposures to the European Central Bank or guaranteed by the European Investment Bank.

Finally, aggregate across single name exposures. In fact, the Market Concentration Risk module will then be computed as the square root of the sum of the squared of the concentration of each counterparty i, which are assumed to be independent between them (i.e., correlation, equal to zero):

$$Mkt_{conc} = \sqrt{\sum(Conc_i^2)}$$

3.1.2 Counterparty Default Risk

Counterparty Default Risk covers the losses arising from the possibility of default of the counterparts for the future 12 months. It includes risk-mitigating contracts (derivatives such as forwards, swaps, etc) and other credit exposures not included previously in the Spread Risk module (for instance, cash at a bank).

Similarly to Equity Risk, exposures are divided into type 1 and type 2 to which different shocks will be applied. The type 1 exposures include undertakings whose counterparty is likely to be rated and which are not diversified. In the context of BPI Vida e Pensões, it includes Currency Forwards and Cash Deposits. On the other hand, type 2 exposures include all other exposures, which are unlikely to be rated and are usually diversified.

The capital requirement for type 1 exposures ($SCR_{def,1}$) is calculated based on Loss Given Default (LGD) of an exposure and the Probability of Default of the counterparty, which is in turn based on its rating or, in the case of counterparts without a rating, on its solvency ratio (See **Appendix 4**).While for Cash Deposits, the LGD will correspond to the market value invested, for Forward contracts it comprises 90% of the market value plus its risk mitigation (difference between the SCR with and without each contract) effect minus the collateral, nonexistent in the case of currency forward contracts (See **Appendix 4**). Then, all type 1 exposures must be aggregated. Nevertheless, the standard model assumes the possibility of three scenarios for this calculation, depending on the variance of the loss distribution (for more details in the variance computation see **Appendix 12**), given by the following formula:

$$SCR_{def,1} = \begin{cases} 3 \times \sqrt{V}, if & \sqrt{V} \leq 7\% \times \sum LGD_i \\ 5 \times \sqrt{V}, if & 7\% \times \sum LGD_i < \sqrt{V} \leq 20\% \times \sum LGD_i \\ \sum LGD_i, if & 20\% \times \sum LGD_i \leq \sqrt{V} \end{cases}$$

Finally, the total SCR Default is the aggregation of the type 1 and type 2 capital requirements, which allows for some diversification benefits:

$$SCR_{def} = \sqrt{SCR_{def,1}^2 + 1,5 \times SCR_{def,1} \times SCR_{def,2} + SCR_{def,2}^2}$$

3.2 Marginal Standard Capital Requirements

The calculation of the overall Standard Capital Requirements for BPI Vida e Pensões' portfolio is important in order to fully understand the main sources of risk for the insurance company. However, a complete analysis should scrutinize even further the main question of where risk comes from and how it can be explained. In this sense, a marginal approach aims to acknowledge the impact of a particular security in terms of SCR and to explain the sources of such SCR in terms of risk exposure, enabling the allocation of the risks and correlations (correlation of the security with rest of the portfolio) to each security in the global portfolio. Additionally, this information may serve as an complementary input for portfolio managers' decisions and change the way managers perceive risk and impact asset allocation, by allowing them to understand which and why particular investments carry greater/lower capital requirements.

Following the methodology of Leblanc (2011), the marginal SCR will take into consideration both the correlation matrix of risk factors; and, extending this previous work, it will also consider the correlation matrix existent in the sub-modules of risk of equity. Given the result arising from this methodology, we will be able to know the contribution of an asset to the total SCR in terms of the value of total assets and the contribution relative to each level of standard capital requirements. As a consequence, the portfolio manager can evaluate a set of securities and analyze the effect of an increment or decline in the amount invested in the level of capital requirements.

For the computation of this marginal effect, l denotes the index for the different risks, while j is the index for the object/security. It is important to notice that a security may be affected by different risks and thus, each risk is composed by the different securities that are affected by it in the portfolio. Consequently, the formula of the Market Value SCR, with l and m being different risks can be given by:

$$MVSCR^2 = \sum_l \sum_m \rho_{lm} MVSCR^l MVSCR^m =$$

$$= \rho_{mm} MVSCR_m^2 + 2 \times \rho_{ml} MVSCR_l \times MVSCR_m + \rho_{ll} MVSCR_l^2$$

where, $MVSCR_l = \sum_{j=1}^{n} MVSCR_l^j$ and ρ_{lm}corresponds to the correlation factor.

Additionally, p_j denotes the weight that each security has on the total market value of assets and the impact that each risk individual risk exposure has on the securities market value. Theoretically, the marginal SCR of j will correspond to the first derivative of SCR with regards to its weight in the overall value of assets (p_j) - contribution per unit of p_j- multiplied by p_j , which is given through the following formula that relates the Marginal SCR (MSCR) with each individual SCR for each risk and the correlation existing between the considered assets:

$$MSCR_j = p_j \frac{\partial SCR}{\partial p_j} = \frac{p_j \sum_{k=1}^{n} p_k \sum_l \sum_m \rho_{ml} SCR_j^l SCR_k^m}{SCR}$$

Proof is shown in **Appendix 5**. In a Solvency II context, this formula takes into account the risks to which the asset is exposed to, accounting also for the correlation existent among both securities and risks. If one adds all the marginal SCR's in a portfolio, the whole SCR is obtained, reinforcing the decomposition effect of this approach. Moreover, the Contribution to the SCR corresponds to the ratio between the MSCR and the overall SCR and it sums up to 1.

4 Hypotheses

Given the main characteristics of the investment portfolio, we test the following preliminary hypotheses:

1. Fitch Ratings (2011) expects a shift in allocation from short-dated Paper to Term Deposits. However, this result will depend on the particular characteristics of each one of the securities included in these asset classes. On the one hand, the effect

on Spread Risk is not known for sure as it depends on the type of asset (for instance, whether the asset is a covered bond, structured note), duration and rating. Commercial Paper in the portfolio tends to be unrated which would lead to a greater variable shock than highly rated securities; on the other hand, Term Deposits with worse credit quality steps face a more significant shock. However, given the strong exposure to some issuers, Term Deposits in the portfolio suffer from a high Concentration Shock. Thus, as the consequences of the shift in allocation are ambiguous, we will test whether the expectations of Fitch Ratings are found in the portfolio.

   (a) In order to do this, we increase the total market value of the portfolio in 5 M€, first allocating it a) only to Term Deposits and b) only to Commercial Paper to discover in which option the impact on the capital requirements is higher. Nevertheless, it is important to consider the limitations of this approach, given that the increase in capital requirements is sensitive to the initial conditions of the portfolio (i.e., pre-existent exposures to Term Deposits, issuers of the Commercial Paper).

2. Given the excessive exposure of Term Deposits in the parent bank BPI (14,58% of the total investment portfolio or a 13,08% excess exposure), Concentration Risk is expected to be high. Thus, a more diluted distribution of the Term Deposits should reduce this exposure and, consequently, the Concentration Risk module and the final amount of capital requirements. This may be tested in several ways. We focus on two that are considered to be more relevant and provide a more accurate analysis.

   (a) One of the options is to reduce the excess exposure in Term Deposits from bank BPI, allocating it to other Term Deposits present in the portfolio. In fact, the portfolio holds term deposits in three other banks (where two of the them already exceed the exposure allowed given its rating). Nevertheless,

it is still important to understand whether a more "diluted" exposure to these banks will lead to lower Concentration Risk charges and, consequently, while accounting for the changes in Interest Rate and Spread risks, to a lower Market Risk and overall SCR. In order to do this, the final excess exposure was set to be equal between the four banks (BPI and the other 3) which is equivalent to say that the market value invested in each one of the entities is equal, given that they have the same threshold exposure (which results in a final excess exposure of 5,40% to each one of the entities). However, there are not only term deposits from these entities, but also corporate bonds. Given that this excess exposure should only be allocated to term deposits, the market value of the remaining securities is maintained fixed while only adjusting the market value of term deposits.

(b) The second option consists in totally eliminating the excess concentration of the BPI's Term Deposits while allocating the corresponding market value to short-dated Commercial Paper from different entities, not exceeding the threshold exposure for each one. Naturally, this is the preferable option in terms of concentration risk given that its excessive exposure is indeed substantially reduced; however, the effects in interest rate and spread risk are ambiguous. Nevertheless, it is important to understand that both term deposits of the bank BPI and the commercial paper to which the exposure is allocated have small and comparable durations (0,08 of BPI against 0,07 average from the securities) such that the results should not be highly affected by it.

3. In this setting, the risk of Default of a counterparty is also important to analyze. The portfolio of BPI Vida e Pensões contains Cash Deposits and Currency Forwards which risk charges are determined in the Default Counterparty Module. In particular, and focusing on Cash Deposits, the risk charges depend on the variance within the securities of each one of the entities included but, at the

same time, on the variance between each one entities (see **Appendix 4** for further details). In fact, depending on variances and the sum of the LGD of the counterparts, different formulas will be used to calculate the final Default SCR. In the original portfolio, given these parameters, the final Default Risk is the sum of each one of the LGD included. However, by changing slightly the exposures to each one of the counterparts or by including additional counterparts the formula applied may be different, which can have significant impact on the final Default Risk. In order to test whether the risk charges for this module can be reduced and having identified a significant exposure to Cash Deposits to the bank BPI, a fictitious portfolio was created. The assumptions are the following: a) Exposure to the bank BPI is reduced by 99M€; b) This amount is allocated to three entities (33M€ each) such that the total exposure in M€ is unchanged; c) The three other entities have the same rating and thus, the same probability of default as BPI, in order to allow for a comparable analysis.

5 Data

To investigate the marginal effect on capital requirements given by Solvency II, the universe of analysis are the insurance portfolios of BPI Vida e Pensões in which the risk is taken by the insurance division.

The composition of the portfolios of BPI Vida e Pensões is taken as of 31th August 2014 as well as its main characteristics such as Market Value, Portfolio ID, Currency, Classification, Cash Flows, among others. Moreover, other data required for the calculation of the SCR is taken directly from Bloomberg, such as Ratings, Duration, Time to Maturity, Funds Composition and their characteristics, etc. As some securities are still quoted in amortized cost, market quotations are taken from the bank's database.

The one-year expected returns used for optimal allocation are also obtained from BPI Asset Management, namely from the Asset Allocation team.

5.1 Portfolio Analysis

The universe of analysis comprises 13 insurance portfolios, with a total market value of 3.820M € and a total of 434 securities. **Table 1** reports the composition of the total insurance portfolio discriminated by sub-portfolios, namely its Market Value, percentage of assets that each portfolio represents relative to the total asset value of the firm and Composition by asset type (i.e., total investment in each asset class).

Table 1: **Portfolio Composition**

Portfolio	MV (M€)	% Total Assets	Equity	Govt. Bonds	Deposits	Corporate	Cash	Mortgages	Fwd	Struct. Notes	Funds	Other
Novo Aforro Familiar	2135,19 €	55,88%	0,0%	19,6%	32,6%	46,1%	2,7%	0,1%	-0,1%	0,0%	0,0%	-0,9%
Reforma Aforro	1139,75 €	29,83%	0,0%	30,2%	16,6%	53,1%	1,1%	0,0%	0,0%	0,0%	0,0%	-1,0%
Aforro Não Residentes	291,20 €	7,62%	0,0%	20,3%	36,0%	26,7%	7,7%	0,0%	-0,1%	0,0%	0,0%	9,3%
Carteira Gestão	172,04 €	4,50%	0,0%	0,0%	0,0%	84,8%	9,4%	0,0%	0,0%	0,0%	5,8%	0,0%
Reforma Garantida	68,45 €	1,79%	0,0%	0,0%	0,0%	93,1%	6,9%	0,0%	0,0%	0,0%	0,0%	0,0%
Plano R	5,44 €	0,14%	0,0%	37,7%	10,1%	33,2%	19,0%	0,0%	0,0%	0,0%	0,0%	0,0%
TG 807	3,65 €	0,10%	0,0%	98,5%	0,0%	0,0%	1,5%	0,0%	0,0%	0,0%	0,0%	0,0%
PPR BBI Vida	2,36 €	0,06%	0,0%	75,6%	8,4%	11,6%	80,0%	0,0%	0,0%	0,0%	0,0%	-75,6%
Carteira Gestão (TIT Negociação)	1,63 €	0,04%	0,0%	0,0%	0,0%	0,0%	84,1%	0,0%	0,0%	0,0%	15,9%	0,0%
PPI	1,00 €	0,03%	0,0%	90,2%	5,0%	0,0%	4,8%	0,0%	0,0%	0,0%	0,0%	0,0%
TG 806	0,11 €	0,00%	0,0%	0,0%	0,0%	0,0%	100,0%	0,0%	0,0%	0,0%	0,0%	0,0%
Sul PPR	0,06 €	0,00%	0,0%	86,4%	0,0%	0,0%	13,6%	0,0%	0,0%	0,0%	0,0%	0,0%
Aforro Familiar	0,01 €	0,00%	0,0%	98,1%	0,0%	0,0%	1,9%	0,0%	0,0%	0,0%	0,0%	0,0%
Total	**3820,89 €**	**100%**										

It shows substantial heterogeneity in the composition of each individual portfolio and total market value, with the 3 biggest insurance portfolios representing more than 92% of the total assets under management. Additionally, there is also disparity in the constitution of each sub-portfolio by asset class: for instance, "Novo Aforro Familiar" is a diversified portfolio, with 20% invested in Government Bonds, 32% in Deposits and 46% in Corporate Bonds while "Reforma Garantida" only holds 93% of Corporate

Bonds and 7% of Cash. These disparities could have significant implications for the capital requirements at the portfolio level, which are relevant for each portfolio manager, however, the computation of total SCR is made at the insurance company level. Thus, this means that each relevant decision for a portfolio manager depends only on the total portfolio in which it is included.

Table 2 reports the composition of the total Investment Portfolio, discriminated by Asset Class. **Column (1)** reports the Total Investment in the Portfolio in Euros and **Column (2)** the same but as a percentage of the total assets of the investment portfolio. The most relevant investments are Corporate Bonds, Deposits (which include commercial paper and term deposits), Government Bonds, Cash and Investment Funds.

Table 2: **Composition Total Investment Portfolio**

	Total Investment Portfolio (in €)	**Total Investment Portfolio (in % Assets)**
Equity	3.699 €	0,00%
Government Bonds	830.587.508 €	21,74%
Deposits	990.680.262 €	25,93%
Corporate Bonds	1.878.617.576 €	49,17%
Cash	117.074.381 €	3,06%
Mortgages	1.552.792 €	0,04%
Forwards	-2.224.781 €	-0,06%
Structured Notes	300 €	0,00%
Investment Funds	10.282.025 €	0,27%
Other	-5.681.599 €	-0,15%
Total	**3.820.892.162,33 €**	**100%**

Analyzing the investment in Corporate Bonds, it is found that the average duration is 0.88 years and it comprises 213 securities. The average CQS of Corporate Bonds is 4,75, which corresponds to a BB rating in the Moody's rating table and about 10% are Covered Bonds. On the other hand, all the existent Government Bonds in the portfolio are from the EEA (Spain, Italy, Germany and Portugal), which means that they suffer a 0% Spread and Concentration Risk factor. Its weighted average duration is 0,42 years, which influences the interest rate risk component. Focusing on Deposits: the Commercial Paper detained by BPI Vida e Pensões has a short duration (average

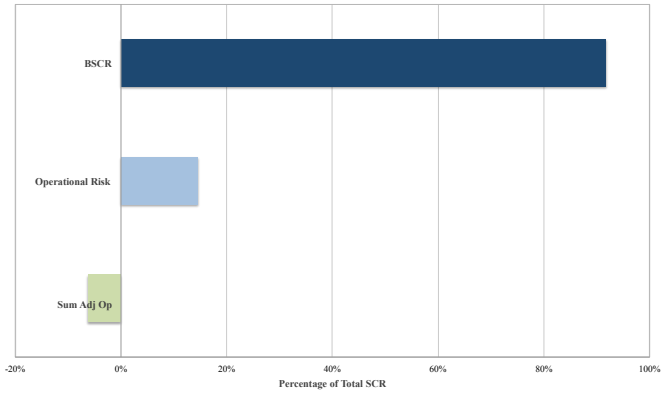
0.09 years) and it is mostly unrated, thus, given the latter we expect that they suffer a high Spread shock; Cash Deposits come mostly from the parent bank (BPI) and thus, it is expected that these securities are severely affected by concentration shocks and the average CQS is 4 (BB Moody's rating). Finally, Term Deposits' average rating is BB with an average duration is 0.14 years. On the other hand, the Investment Funds contain both fixed income and equity securities, to which the lookthrough approach is used. Finally, Equity present in the portfolio is composed only by Type 2 Exposures (outside the EEA or OECD), in which an alternative fund "Imofomento" is included. Finally, there is a small percentage of Structured Notes and Currency Forwards in the portfolio, whereas the latter is used to cover the currency exposure of a specific corporate bond denominated in USD dollars.

6 Results

6.1 Total SCR

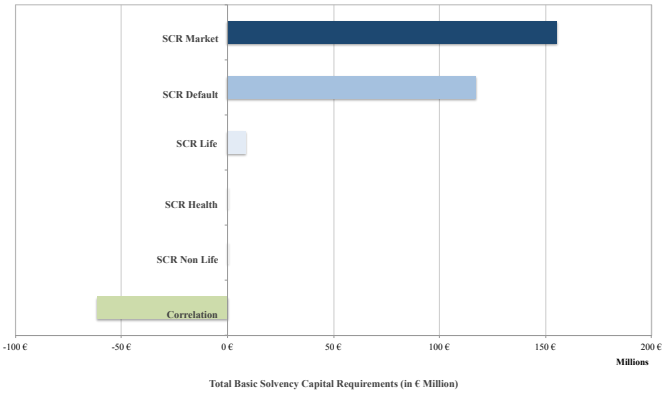
Following Solvency II Standard Formula methodology presented, the results show that the value of the total Standard Capital Requirement for the company is of approximately 239,43M€ by 31st August 2014. By observing **Figure 1,** it can be seen that most of the SCR is explained by the Basic Solvency Capital Requirement (BSCR) with a 92% contribution to total SCR (or 219,56M€). Operational risk contributes with 14,5% (34,74M€), while the negative effect of the adjustment for the risk absorbing effect of technical provisions and deferred taxes of 14,86M€ reduces total SCR (given by discretionary benefits).

Figure 1: **Total Standard Capital Requirements**



Considering the importance of the BSCR in the company's capital requirements, it is important to identify its main sources. **Figure 2** shows that Market and Default Risk are the main sources of BSCR (155M€ and 117M€), reaching a total value, after correlations, of 216,66M€ or 98,68% of total BSCR. Life Underwriting risk explains the remaining 1,32%.

Figure 2: **Total Basic Solvency Capital Requirements**



Consequently, it can be seen that capital requirements are mainly driven by the asset side, implying that, if the portfolio manager aims to manage this requirements, he should pay close attention to the type of assets in the portfolio. Furthermore, this value includes the diversification effects given the non-perfect correlation among securities, which allow for the insurance company to "save up" to 122,17M€ (considering correlation factors both within Market Risk and between this and Spread and Life Risks), calculated by the difference between the sum of the BSCR for all securities and

the value obtained employing EIOPA's correlation matrices. Later, we will evaluate the level of sensitivity of the BSCR to the correlation matrix used in the calculations and, thus, how diversification can have a leading role in the capital requirements.

Therefore, the evaluation of the asset-side effect on capital charges is essential. In the Solvency II framework, cash deposits and risk-mitigating contracts such as forwards are included in the Counterparty Default segment, while other assets are assessed in the Market Risk module. The following analysis will focus on these modules and how the EIOPA specifications can be applied to the portfolio.

6.2 Counterparty Default Risk

As mentioned, in a BPI Vida e Pensões context, Default Risk includes Cash Deposits and Forwards, both considered to be Type 1 exposures. Cash Deposits have an average rating of BB and average probability of default of 1.66%, with a clear bias towards the bank BPI, which concentrates nearly all cash deposits. For these securities, the loss given the counterparty defaultl fully corresponds to their market value, while for the Forwards it is 90% of the market value plus its risk mitigation effect minus the collateral, inexistent in the case of currency forward contracts. Given the intermediate calculations for $V_{intra}$and V_{inter}(**Appendix 4.2**), the final amount for capital requirements will be the straightforward sum of the LGD of all securities, resulting in a total of 117M€, where, if one considers the original market value, corresponds to a shock close to 100% of the market value, as it is assumed that all of the value invested in this assets would be lost in the event of a default.

As it will be seen in the breakdown of the capital charges by individual security, Cash Deposits play a very important role in driving up the SCR as nearly all of its market value should be intended as LGD and, consequently, be placed under capital requirements, especially given the high variance of the loss distribution present in the portfolio. As seen, Hypothesis 3 aims to access the sensitivity to the SCR not only to this exposure but also to the underlying variance of the portfolio.

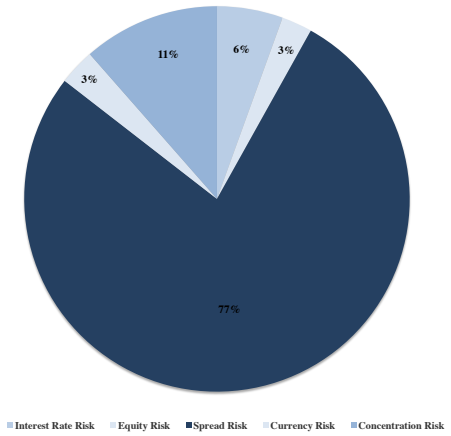
6.3 Market Risk

Alongside Default, Market Risk has a great impact in the total SCR of the company. Preliminary results show that the Market Capital Requirement for Solvency II purposes, i.e. taking into account the correlation among risk classes, is approximately 155,38M € (or 4,07% of Total Assets), while its absolute value assuming perfect correlation among risks can sum up to nearly 217M € (5,68% Total Assets). These results imply a gain of 61,55M€ which can be attributed to diversification effects as capital requirements are lower given the non-perfect relationship between the risks considered.

6.3.1 Decomposition of the Market Risk

It is crucial to know what are the main drivers of Market Risk as different types of assets are exposed to different risks and, consequently, to different shocks and capital requirements. The results in **Figure 3** show that Spread Risk is the main contributor, followed by Concentration Risk, Interest Rate, Currency and, lastly, Equity. Considering the final Market SCR value, Spread and Concentration take nearly 88,9% of the whole value (77,4% and 11,5%, respectively) and the remaining is distributed as follows: 5,5% come from Interest Rate, 3,03% from Currency and 2,6% from Equity.

Figure 3: **Market Risk Decomposition**



These results may seem odd at a first glance as Equity and Currency tend to be

the sub-modules that experience a greater shock over market value. However, it is crucial consider the portfolio composition in order to address this analysis. Indeed, the portfolio holds a residual investment in equities (3.699€), being that Equity Risk capital charges (3,40M€) are mainly explained by "Imofomento" (a 10M € alternative investment regarded as equity) and look-through in investment funds. Additionally, BPI Vida e Pensões' portfolio has a large fraction of fixed income securities, a total of 96,84% including Corporate and Government Bonds as well as Term Deposits and Commercial Paper,which justifies the influence of Spread and Interest rate risk. Concentration risk, not linked to portfolio composition but to excessive exposure to particular counterparts and independent from other risks, is also one of the greatest contributors for SCR, mainly due to a over exposure to the own bank (BPI) and other portuguese entities. While most of the excessive positions do not tend to exceed 5% relative to its threshold, Banco BPI has a 14,6% weight in the portfolio, 13,1% over the EIOPA stipulated threshold of 1,5% .

However, the overall capital requirements does not correspond to a simple sum of the risks which given imperfect correlation could provide an excessively conservative valuation and overestimate the capital charges for the company. In fact, the differences between the values achieved and the ones assuming perfect correlation scenario are considerable, which suggests a great sensitivity from SCR to the correlation matrix. Therefore, the following section aims to test how the capital requirements varies with changes in the correlation matrix, namely the one used for Market Risk.

6.4 Diversification

The assumption that all asset classes are not perfectly correlated among each other is taken into account in the Standard Solvency II framework by the use of correlation matrices between the different types of risks to which the firm is exposed to. This assumption, which reflects the markets' behavior since risk movements are not perfectly aligned with movements in other classes, allows for a reduction in the overall Basic SCR

of around 122 M€. Thus, it becomes clear that the total amount of capital requirements is quite sensitive to the correlation matrices used for the different modules and sub-modules.

Consequently, in this section, we aim analyze the impact of small changes in the correlations' assumptions in the total capital requirements. Thus, the value of BSCR as well as the diversification gains are computed using different correlation matrices, based on four different assumptions: a) all risks are uncorrelated, which is expected to give us the lowest value for capital charges; b) all risks are perfectly correlated (which is equal to the value before correlations are accounted for, that is, a simple sum of SCRs already computed); c) a 5 percentage points increase in all correlations and d) a 5 percentage points decrease in all correlations. The results are showed in **Table 3.**

Table 3: Scenario Analysis: Correlations

	Original Portfolio	**Zero Correlation Portfolio**	**5 p.p. Downward Shock**	**5 p.p. Upward Shock**
SCR Market	155.383.497,52 €	141.177.919,88 €	153.794.249,63 €	156.956.745,48 €
SCR Market (%)	4,07%	3,70%	4,03%	4,11%
SCR Default and Market	216.659.200,62 €	183.422.317,96 €	211.084.160,46 €	222.175.546,54 €
SCR Default and Market (%)	5,67%	4,80%	5,52%	5,81%
BSCR	219.559.402,17 €	186.646.806,22 €	214.043.784,31 €	225.019.912,09 €
BSCR (%)	5,75%	4,89%	5,60%	5,89%
Gains from Correlation	122.172.509,23 €	155.058.326,53 €	127.688.127,09 €	116.712.052,28 €

Starting with the independence between classes of risk, in **Column (2)**, the total value of BSCR is reduced from 219,56M€ to 186,65M€, with diversification gains increasing by more than 25% when compared to the original output (total of 155M€). Moreover, and considering the results arising from a perfect correlation scenario, it can be seen that diversification effects not only account for a great reduction in capital charges but that these effects are extremely sensitive to the correlation matrices that are considered. In fact, the total BSCR under perfect risk correlation is equal to

341,7M€, well above any of the charges previously presented.

The level of sensitivity towards the correlation matrices used is also reinforced when considering a up and downward shock of 5 percentage points in the correlation factors between risks (**Column (3)** & **(4)**). As expected, capital requirements increase when assuming greater inter-dependence among risks and decrease in the downward state, being that the changes against the original computations are similar in absolute terms and around 5,5M€, leading to a final SCR of 5,89% in the upward state and 5,60% in the downward one. Thus, it can be concluded that even small changes in the correlation matrices will lead to considerable change in the value of the Basic Solvency Capital Requirements and in the diversification gains.

Consequently, and as it would be expected given the reduction in SCR with the use of EIOPA's correlation matrices, the overall values of capital requirements are quite sensitive to the assumed dependence between classes of risk, promoting portfolio diversification. In fact, the less conservative the relation between risks, the higher are the diversification gains arising for the portfolio in hands. In opposite scenarios, BSCR may vary between 341,7M€ and 186,7M€. Therefore, the choice between alternative correlation matrices needs to be well considered as the capital charges achieved may lead to two opposite consequences regarding the true necessities the company has: if the matrices are too conservative, capital requirements may be overestimated; however, if they are not conservative enough, the firm may set apart insufficient capital to face future necessities. This reasoning needs to be done not only by EIOPA but also when developing internal models which, in an attempt to minimize capital charges, may take correlation matrices that poorly reflect the relationship between risks. Additionally, this section also provides insights over the importance of holding a well diversified portfolio. As classes of assets are not perfectly correlated, the manager can face lower capital requirements by having securities spread over the different risk classes. In this sense, BPI Vida e Pensões' portfolio, regardless of its fixed income focus, already is somewhat diversified, which leads to capital charges benefits.

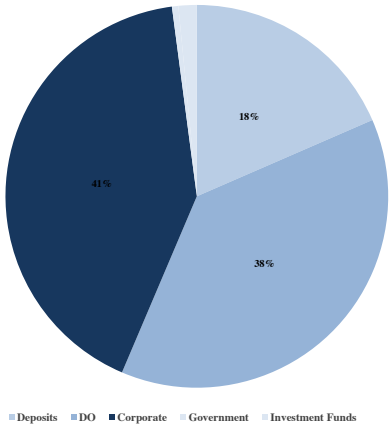
6.5 Marginal

Applying directly the standard formula by EIOPA allows the company to know the total capital requirements and the distribution between the main risks to which it is exposed to. Nevertheless, a more in-depth approach could help the managers to better understand where risks come from and which type of securities are more exposed to a particular risk class. By knowing that, they are able to manage their portfolio with a greater awareness to the risk they are actually incurring into. Consequently, this section analyzes the marginal contribution by asset class to total risk, discriminating afterwards into securities and also providing insights over the portfolio distribution of the capital charges. In this sense, marginal contributions of each security and class will depend on both the type of asset in question, but also on the correlation between the modules of risk.

6.5.1 Marginal Contribution by Asset type

Regarding the Basic Solvency Capital Requirement, it can be seen that most of the capital charges can be attributed to three asset classes: Corporate Bonds, Cash Deposits (DO) and Term Deposits as observed in **Figure 4.**

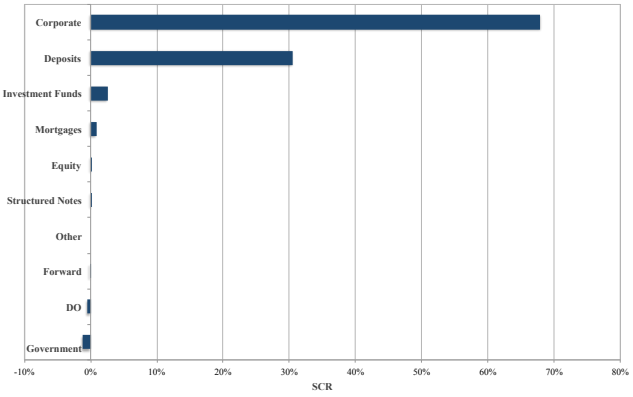
Figure 4: **Marginal SCR by Asset Type**



With a contribution of 41%, 38% and 18% of the total amount, respectively, they explain 98,53% of the total value, while the remaining charges come from Investment Funds and Mortgages. On the other side, Government Bonds holds a slightly negative contribution given the very low capital capital requirements, more than offset by the liabilities effect.

Focusing on Market Risk, which represents 4,07% of total assets, **Figure 5** shows that it is explained by Deposits and Corporate Bonds with a 30,47% and 67,85% contribution, respectively, consistent with the high concentration of fixed income securities in the portfolio. Government Bonds have a -1,18% contribution and Investment Funds explain 2,58% of the total charges.

Figure 5: **Market Risk Contribution by Asset Class**



To better understand the reasons for the relative importance of each asset class, we sort the top 150 securities by contribution (since many securities have residual exposure, the liabilities distribution, even though allocated with regard to their portfolio weight, may lead to values that would bias the results). The top 150 securities explain most of the SCR, and their marginal contributions are decomposed into the different classes of risk that contribute to that result. By doing so, one can know in-depth where the risk of one security comes from and, on average, what are the leading risks for each one of the asset classes.

For instance, it is expected that Term Deposits and Commercial Paper will be highly exposed to Spread Risk with little exposure to Interest Rate Risk and none to Equity

Risk. Moreover, especially for Term Deposits from the bank BPI, the exposure to Concentration Risk is expected to be high. **Table 4** segregates the average marginal contribution of these classes into risks.

Table 4: **Average Marginal Contribution: Top 150**

	Interest Rate Risk	Equity Risk	Spread Risk	Currency Risk	Concentration Risk
TERM DEPOSITS	-10,00%	0,00%	89,77%	0,00%	20,23%
BONDS	39,70%	0,00%	57,73%	0,70%	1,87%
INVESTM. FUNDS	0,00%	69,44%	20,13%	10,00%	0,43%
COMMERCIAL PAPER	-29,53%	0,00%	129,52%	0,00%	2,64%

Term Deposits have a negative exposure to Interest Rate Risk (-10%), arising, once again, from the liabilities effect in this sub-module and high Spread and Concentration Risks (89,77% and 20,23%, respectively), showing no exposure to Equity and Currency risks. Commercial Paper follows a similar pattern, only with some differences when compared to Term Deposits. First of all, as the maturity of Commercial Paper tends to be lower, the negative contribution from Interest Rate Risk is aggravated (lower asset exposure to match the liabilities one); additionally, our portfolio's basket of Commercial Paper heavily relies on unrated companies, which face greater shocks than well rated companies regarding Spread risk, making their average contribution surpass 100% (129,52%). Corporate bonds are expected to show a strong vulnerability to Spread and Interest Risk. The results show a 97,43% contribution from Interest Rate and Spread Risk (39,70% and 57,73%, respectively), while the remaining is explained by a 1,87% exposure to Concentration and 0,70% to Currency risk. Moreover, Equities are mainly driven by Equity and Currency risk (79,48% and 17,88%), as it would be predictable due to the characteristics of the strategic participations held and a 2,64% exposure to Concentration Risk given that these positions are directly linked to BPI. Lastly, and aggregating the characteristics of both fixed income and equities, there are the Investment Funds, which are explained mainly by Equity, Spread and Currency risk (69,44%, 20,13% and 10%).

From this analysis, and due to the inherent characteristics of the securities, a port-

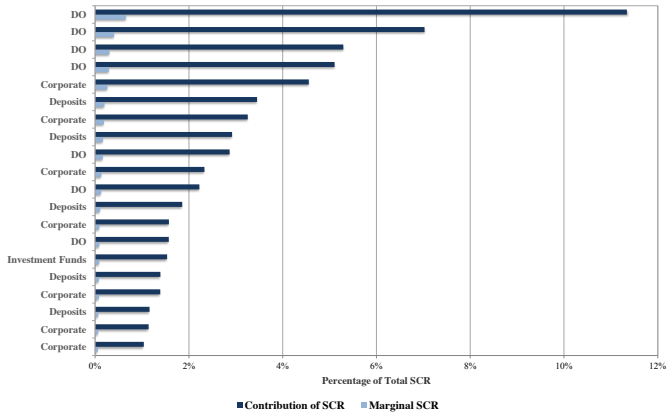
folio manager should rely its asset allocation decisions in the results obtained, moving towards classes with lower capital requirements and away from those with higher charges. However, this may lead to incorrect decisions, especially when not taking into account diversification benefits and Asset Liability Management (ALM).

6.5.2 Marginal Contribution by Security

The evaluation of capital requirements based on the asset class gives the insurer a more complete scope risk decomposition. However, as securities within each asset class have different characteristics, an analysis at security level is also relevant in this work.

The analysis is focused on the top 20 assets, which contribute more for the capital requirements and represent 63,04% of total SCR. The results show that the trend seen in the previous section still holds as 7 out of the 20 securities are Cash Deposits (DO), 7 Corporate Bonds, 5 Term Deposits and one Investment Fund (**Figure 6**).

Figure 6: **Marginal SCR by Individual Security**



The reasons why these securities stand out differ depends on the asset class: for instance, Cash Deposits face a capital requirement that is equal to their market value, thus, the Cash Deposits with the greatest market value from the portfolio are part of top 20 given that most securities face a shock considerably below their own market value. Additionally, Term Deposits, namely the ones from BPI, have the highest market values

among the securities, making them more suitable to rank top in terms of risk. However, this is not the only reason: in fact, these securities face high capital requirements arising from Concentration Risk, other than Spread Risk. Therefore, even when comparing with securities with similar market values, these would always face higher capital charges due to their contribution for the great concentration exposure of the portfolio to the bank BPI; in fact, around 40% of their individual SCR comes from this risk. Regarding Corporate Bonds, the high rank of the respective securities arise from a combination between high market value positions with particular specification that each holds. These specificities are related to high cash flows and relatively high maturities held in big sub-portfolios where liabilities are allocated in a more diluted way, leading to higher interest rate risk; additionally, the securities were either below investment grade ratings or unrated, harshening the shocks applied to the market values. Lastly, the alternative investment "Imofomento" is also one of the constituents of this top, which is related to the high market value subjected to a Type 2 equity shock (of 54% of the market value). As for the analysis of the Top 150, a similar pattern is found in the Top 20, showed in **Table 5.**

Table 5: **Top 20**

	Interest Rate Risk	**Equity Risk**	**Spread Risk**	**Currency Risk**	**Concentration Risk**
TERM DEPOSITS	-14,02%	0,00%	72,52%	0,00%	41,50%
BONDS	35,55%	0,00%	59,42%	4,61%	0,43%
INVESTM. FUNDS	0,00%	100,00%	0,00%	0,00%	0,00%

Term deposits are explained by Spread and Concentration, while negatively impacted by Interest Rate Risk (72,52%,41,50 and -14%, respectively). Corporate Bonds, given their high and frequent cash flows and ratings, see their marginal contribution explained mostly by Spread and Interest Rate Risk (59,42% and 35,55%, respectively). On the other end, both Investment Funds and Cash Deposits are fully explained by Equity and Default risk, respectively.

It is concluded that each security in the portfolio will have a different impact in the capital charges, even within the same asset class. However, not only the market value

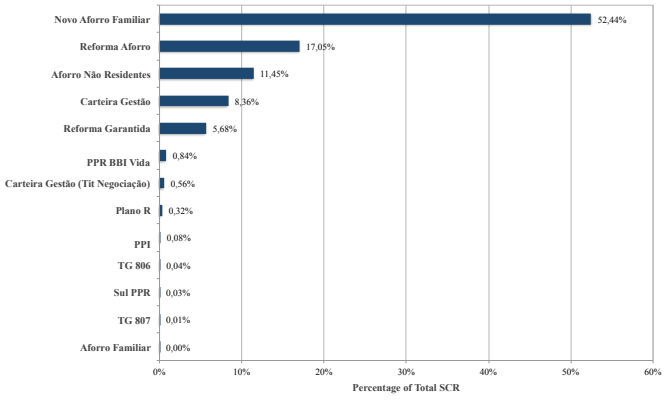
invested will be the reason for this differentiation; but also special characteristics of the asset itself will play an important role, mainly regarding maturity, exposure and credit quality step of the issuer and the existence, or not, of derivatives designed to cover risks such as currency risk in the portfolio in hands.

6.5.3 Marginal Contribution by Portfolio

After analyzing the capital requirements based on a asset class and, more specifically, a security perspective, it becomes relevant to also address the subject at a portfolio level. Since the portfolio is constituted by 13 sub-portfolios, presented previously, it is important to BPI Vida e Pensões to access which portfolios have the largest capital consumption and if it is in accordance with the weight of the overall portfolio. Furthermore, the analysis will also focus on the relation between capital charges and the sub-portfolio's constitution.

Given the diverse market value allocation of each sub-portfolio, it is expected that a similar pattern is found in the capital requirements charges. As observed in **Figure 7** "Novo Aforro Familiar", "Reforma Aforro" and "Aforro Não Residentes", which account for 93,33% of total market value, contribute to 80,93% of the company's SCR.

Figure 7: **Marginal SCR by Portfolio**



For these portfolios, the most representative of the whole set of securities, the weight of both SCR and market value over the respective total amounts does not change much, showing that market value holds a significant importance in determining capital charges.

On the other hand, these portfolios are the ones where capital requirements constitute a lower percentage of market value (6,8% of the value of assets on average). Thus, at a portfolio level, the analysis show that the greatest portfolios are responsible for the largest consumption of capital but, at the same time, are the ones with the lowest SCR as a percentage of assets. This happens mainly due to the number of securities and asset classes, which promotes diversification gains and provides evidence that holding portfolios highly concentrated may not be advisable as such gains may be lost.

In addition, the premise that different asset classes will have different capital charges is reinforced when evaluating the SCR at a portfolio level. Comparing the weight of each asset class in terms of market value and of capital requirements, one can conclude that small positions in Cash Deposits will have a great impact (greater than its original weight in the portfolio) in the final amount of capital. For instance, "Novo Aforro Familiar" holds 3% in cash and this investment explains 31,88% of the total SCR for this portfolio, while "Reforma Aforro" and "Aforro Não Residentes" show a much clearer example: a 1% and 9% investment in cash represent 22,3% and 62,39% of total capital, respectively. On the other hand, the relation between market value and capital requirement seems to be more even for deposits and corporate bonds, yet depending on the type of assets held in each portfolio.

6.5.4 Concluding Remarks

The results obtained from the application of EIOPA Formula show that the main source of risk for an insurance company arises from the asset side. Unlike Solvency I, the type of securities that are held as well as the level of diversification of the portfolio play a crucial role in managing capital requirements, given the different treatments that are given to each asset class.

For the case of BPI Vida e Pensões portfolio, mainly composed by fixed income securities, the main sources of risk arises naturally from the investments in Term Deposits, Corporate Bonds and Cash Deposits. Additionally, it also becomes clear that,

more than its asset class and market value, capital requirements of each individual security depends substantially on the particular characteristics such as issuer's rating and exposure and maturity of the asset.

Though asset classes differ in terms of capital charges, portfolio concentration over a particular class is not advisable. Even though some adjustments to the portfolio could be done to reduce the capital required, as it will be tested further, it is crucial that the overall portfolio remains diversified in order to enjoy diversification gains. Finally, the choice of the correlation matrices is also critical for the level of capital charges as the overall value is very sensitive to changes in this matrices.

The following steps of the thesis will be based on the analysis of the hypotheses previously stated, as well as developing the subject of optimal asset allocation for the particular case of BPI Vida e Pensões.

6.6 Hypotheses Results

1. To pursue this hypothesis, it was assumed a 5M€ increment in the total market value of the portfolio by increasing a) the allocation to Commercial Paper and b) to Term Deposits. As observed in **Table 6** (see below), the SCR as a percentage of assets remains nearly unchanged as the change in the portfolio is not high given the size of the whole portfolio (increases to 6,2700% and 6,27441%, respectively, from 6,2667%). However, in absolute terms, it can be seen that, while both capital requirements increase, a 5M€ additional investment in Term Deposits brings a SCR higher by 166.000€ when compared to the Commercial Paper assumption. This is achieved given the higher capital requirements on Interest Rate, Spread and Concentration of the former, which goes in accordance with the characteristics of both asset classes: Term Deposits have a higher duration (0,14 years versus 0,09 from Commercial Paper), suffer higher spread shocks (unrated securities have lower shocks than poorly rated entities) and are also more exposed to Concentration Risk as it was already presented. Thus, the prediction of Fitch

ratings that a shift from Term Deposits to Commercial Paper occurs is not valid in the portfolio at hands, especially given the specific attributes of both asset classes.

2. Focusing now on Hypothesis 2 (see **Table 6** below):

   (a) By pursuing this hypothesis, the final excess exposure to each one of the four entities considered is 5,40% of the total asset values. The results show a considerable reduction in Concentration Risk, from 1,38% of the total assets in the original scenario to 0,37% before taking into account correlations inside the market risk module. Nevertheless, both interest rate and spread risk change as the market value invested in the term deposits must be adjusted as well. In fact, after the adjustment, Spread Risk increases significantly given that the market value of securities with a higher duration and with a worse credit quality step than BPI has increased (two out of the three other banks have an CQS of 5 instead of BPI's CQS 4), thus they will suffer a higher shock. On the other hand, the average duration of the term deposits of BPI is 0,08 while the average duration of the other securities is 0,20. The same happens for Interest Rate Risk: the risk increases for the securities in which the market value increased and vice-versa. Overall, the total Market Risk of the insurance company goes down from 4,07% of total assets to 4,05%. Nevertheless, the total SCR increased from 6,27% to 6,37% as, regardless of the reduction in Market Risk, the adjustment for loss absorbing effect of technical provisions, dependent on the value of the total BSCR is also reduced, leading to a lower negative effect in the overall SCR. Nevertheless, it is important to take into consideration that the allocation of the term deposits was not optimal: in fact, if the allocation were to be to securities with an approximate duration and the same credit quality step as the parent bank BPI, although the Concentration risk charge would be the same, Interest Rate and Spread Risk would be less penalized, allowing for a

reduction in SCR market and, consequently, on the total SCR.

i. We explore this idea by creating a new fictitious portfolio (**Table 6**) in which all term deposits have the same credit quality step (CQS=4) and the only change between the initial and final portfolio is therefore the initial and final market value of the Term Deposits and thus, its concentration exposure. Once again, the final excess concentration of all of the Term Deposits is set to be equal between each other. Given that in these fictitious portfolios all of the securities have the same CQS, the initial and final Spread Risk will remain approximately the same. Also, the Interest Rate charge remains approximately constant and thus, it allows to understand the true impact of reducing Concentration Risk in the total SCR of the insurance company. In fact, as shown in **Table 6**, Concentration Risk is reduced from 1,38% to 0,37% and as the Interest Rate and Spread Risks remain almost unchanged, the Market Risk charge decreases substantially (from 3,98% to 3,75% of the value of total assets). Finally, the total SCR is reduced from 6,19% to 6,13%, which implies that reducing Concentration Risk by distributing the allocation to securities from different issuers can significantly impact the capital requirements of the bank.

(b) In order to eliminate completely the excess exposure of Term Deposits from the bank BPI, the allocation of the market value was performed to 13 different entities with short-term dated Paper. As referred, the average duration of the term deposits from BPI is 0,08 while the average duration of the other entities is 0,07, not representing a significant difference. Nevertheless, Concentration Risk is significantly reduced which dominates the reduction in Market Risk (from 4,07% to 3,69%) and thus, in the total SCR. The final SCR of the fictitious portfolio represents 6,09% of the assets, down from 6,27%. Thus, as expected, this is the hypothesis that provides the biggest re-

duction in the capital requirements of the insurance company. To conclude, both concentration's analysis performed show how important Concentration Risk is and how a well-diversified portfolio may considerably reduce SCR, especially when taking into account the specificities of the securities to which the allocation is preferred.

3. Given that Default Risk is computed separately from Market Risk, the latter does not change, while the former decreases significantly, from 117M€ to 72M€ (a reduction of 38%). Finally, after taking into account the correlation between the Market and Default modules, the total SCR decreases to 5,57% of the assets value. This test shows that optimizing Default Risk should also be very important for the insurance company as it allows for substantial reduction in total SCR.

Table 6: **Summary of Results from Hypotheses**

Original Portfolio		**Portfolio 1. a)**	**Portfolio 1. b)**	**Portfolio 2. a)**	**Portfolio 2. b)**	**Portfolio 3.**	**Fictitious Original Portfolio**	**Fictitious Portfolio 2.a)**
	% Assets	**% Assets**					**% Assets**	
BSCR	5,75%	5,75%	5,75%	5,73%	5,43%	4,97%	5,67%	5,48%
Adjusted SCR	-0,39%	-0,39%	-0,39%	-0,27%	-0,26%	-0,31%	-0,39%	-0,27%
SCR Op	0,91%	0,91%	0,91%	0,91%	0,91%	0,91%	0,91%	0,91%
SCR	**6,27%**	6,27%	6,27%	**6,37%**	**6,09%**	**5,57%**	**6,19%**	**6,13%**

Market Risk		**Market Risk**					**Market Risk**	
	% Assets	% Assets					% Assets	
Interest Rate	0,40%	0,40%	0,40%	**0,41%**	**0,41%**	0,40%	0,40%	0,41%
Equity	0,14%	0,14%	0,14%	0,14%	0,14%	0,14%	0,14%	0,14%
Spread	3,38%	3,39%	3,39%	**3,59%**	**3,24%**	3,38%	3,29%	3,29%
Currency	0,37%	0,37%	0,37%	0,37%	0,37%	0,37%	0,37%	0,37%
Concentration	1,38%	1,38%	1,38%	**0,37%**	**0,21%**	1,38%	1,38%	0,37%

	%BSCR	%BSCR					%BSCR	
SCR Market	70,77%	70,79%	70,83%	**70,63%**	**68,00%**	81,80%	70,16%	68,47%
SCR Default	53,32%	53,29%	53,24%	53,47%	56,39%	**38,12%**	54,00%	55,88%
SCR Life	3,57%	3,97%	3,97%	3,99%	4,20%	4,59%	4,03%	4,17%
SCR Health	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
SCR Non Life	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Correlation	-28,06%	-28,06%	-28,05%	-28,09%	-28,59%	-24,51%	-28,19%	-28,51%

7 Optimal Asset Allocation

The analysis of capital allocation strictly based on the capital charges inherent to the portfolio does not take into account the risk-return trade off faced by portfolio managers. Typically, asset classes that have higher risk are the ones yielding greater returns, as increments in return are usually interpreted as a compensation for the undertaking of a higher amount of risk. In this sense, it is possible to extend our analysis to a risk-return framework, evaluating how the portfolio manager could optimize the assets held in the portfolio with regards to the expected return targeted. Nevertheless, the definition of risk used is not the usual standard deviation but instead it represents the capital charge of the portfolio, the SCR, which is by itself a risk-sensitive measure. As this optimization can be performed for multiple expected returns, a broader study can be conducted to understand how the risk of this portfolio varies with constraints over the expected return. In fact, this analysis will be performed over a set of portfolios that minimize the SCR for a given level of expected returns; thus, the outcome constitute our portfolio's efficient frontier i.e. set of optimal portfolios that offer the lowest risk for a given level of expected return, considering the constraints imposed to assure feasibility and portfolio characteristics.

In order to reach the expected return of the portfolio, the 1- year expected return per class of assets is taken as an input and obtained from BPI's Asset Allocation team. Moreover, a 0% return was assumed for Cash, while for Deposits, including

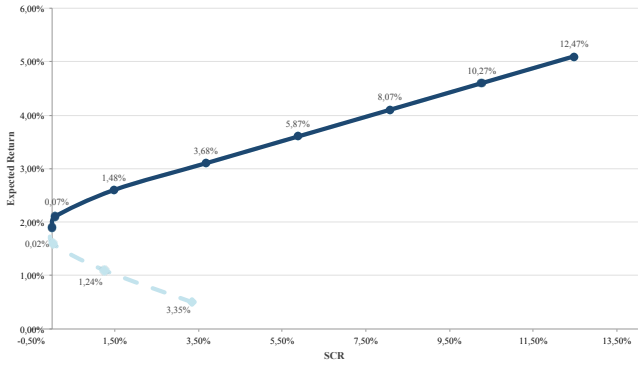
Term Deposits and Commercial Paper, it was considered the 1-month EURIBOR as of 31st August 2014 rate as most of the securities in our portfolio hold a duration close or lower than one month. Expected return for each of the assets classes considered can be seen in **Appendix 6**. As it is expected by the risk-return trade-off theory, return is directly related to the level of risk of the asset, as Equities, the riskiest class, yields greater returns while Fixed Income securities, assumed as securities with low or no risk, present lower returns. The total portfolio return computed given these assumptions is 1,6%, highly influenced by the Cash and Deposits' weights in the portfolio.

For the computation of the efficient frontiers three scenarios are set, differing only on the kind of restrictions imposed. In Scenario 1, it is assumed that the portfolio manager can construct its portfolio at will, taking only into account a few restrictions: a) sum of weights in terms of market value in the portfolio must equal 100%; b) no short positions can be undertaken in any class and c) expected return is equal to the one obtained with the current constitution (1,6%). Overall, in Scenario 1 (See **Figure 8**), SCR falls considerably from 5,75% to 0,02% of total assets. The results show a clear focus on Eurozone Bonds, investing 89,31% of the total value of assets in this class. This is due to the fact that, due to EIOPA specifications for capital charges, Eurozone Bonds are merely exposed to Interest Rate Risk. However, when accounting for both the asset and liability side of such risk, it can be seen that the liability side exceeds the asset side, resulting in an overall negative SCR for this set of securities. Furthermore, given the low expected return of the portfolio, it is easy to satisfy the return constraint since, by itself, the Eurozone Bonds class yields 1,7% which is already higher than the 1,6% imposed. Regarding the other classes, the amount invested in Deposits is reduced (from 25,93% to 10,59%), while positions in Corporate bonds and Cash are nearly set to 0. Positions in Equity are also slightly increased, working as a way to leverage expected return (a small increment in exposure can have a significant impact in the expected return given their greater asset return).

In order to reach the efficient frontier, the same logic was applied to different levels

of expected returns, starting off at 0,5% and ending at an expected one-year return of 5,1%. As **Figure 8** suggests, the SCR increases as demand for expected returns is higher, but SCR also increases when it is lower than the current 1,6% level.

Figure 8: **Efficient Frontier - Scenario 1**



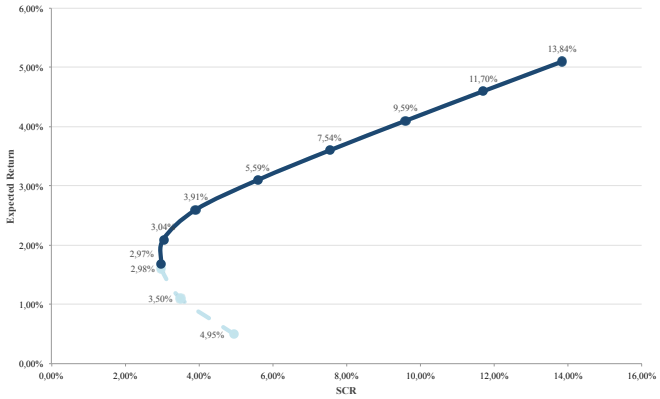
Therefore, for the same level of risk, expected return may vary, which supports the belief that some portfolios are more efficient than others given that it is possible to choose a portfolio with a lower capital charge but with similar returns. In this case, by setting the expected return to a low value, the manager is obliged to invest in securities with lower return but that are heavily charged in terms of risk, which happens with Deposits. Indeed, as expected return increases, the investment in deposits (74,50% for a 0,5% return) decreases to 0% (given that it provides a low return for the level of risk it entails) - See **Appendix 15** for allocation bets in each scenario. The investment in Eurozone Bonds goes in the opposite direction, being the privileged bet for optimization; however, when the bar for return is set higher than its own asset return, the optimal portfolio shifts direction towards Peripheral bonds, given that they are still subjected to low charges and yield a higher return. In fact, for a 2,6% return, investment in this class sums up to 93,08%, leveraged by a 7,89% position in the Equity strategic participations held in the portfolio. From then onwards, the distribution between Bonds and Equities changes as return requirements become more exigent. For the last portfolio tested, the optimal security set is constituted by 59,97% in strategic Equity participations and 41% in Peripheral Bonds, reaching an expected return of 5,1%.

In short, portfolio composition will change considerably depending on the expected level of return. However, some of the tested portfolios are not efficient and should not be chosen as there are other portfolios that lead to the same risk at a higher gain. Indeed, this frontier starts at the so-called minimum variance portfolio, the point in which risk (SCR) is minimal, and which is most likely to correspond to a nearly 100% investment in Eurozone Bonds. Only to the right of the minimum variance portfolio onwards the portfolios are efficient and, thus, constitute the optimal asset allocation for a particular return.

As asset managers are often subjected to constraints over the minimum amount to hold in certain asset classes, testing this possibility is important, not only to increase the realism of the analysis but also to serve as a term of comparison to the first scenario already presented.

Thus, in Scenario 2 a liquidity restriction is placed, assuming that the bank may be interested in keeping a fixed level of liquidity/cash in order to fulfill potential situations in the short-term. For this case, a 3,01% in Cash will be kept given that it is the current allocation in the portfolio. **Figure 9** shows a clear shift of the efficient frontier to the right, implying that keeping liquidity will limit the ability of the manager to reduce substantially the capital charges, especially given the high capital requirements that Cash Deposits are subjected to in the Default Risk module.

Figure 9: **Efficient Frontier - Scenario 2**

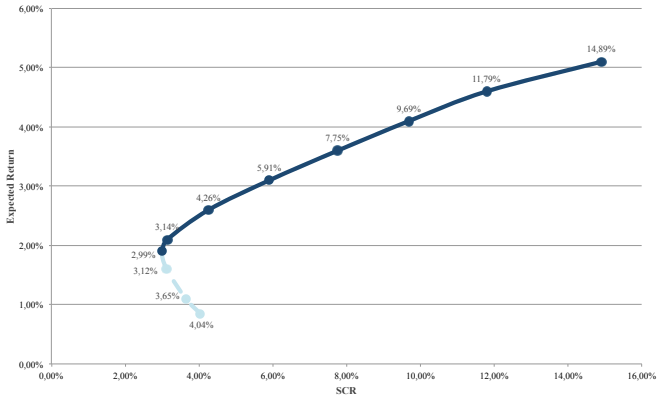


Consequently, while in Scenario 1 a 1,6% expected return could be achieved with

a 0,02% SCR under no class limitation, it will now require a SCR of 2,98% of the company's assets. However, the preference for Eurozone Bonds is maintained. For the remaining curve, a similar trend is exhibited for this scenario, being the shift firstly towards to Peripheral Bonds and latter Equities; while the shift to equities is now more significant as holding 3% in Cash is equivalent to earning no return. Therefore, a shift towards more profitable asset classes is necessary which increases also the capital requirements (while for the previous case the maximum SCR was of 12,47%, it is now of 13,84%).

In Scenario 3, on top of the liquidity restriction, one places a 50% cap per asset class in order to promote some diversification in the portfolio and making it less sensitive to sudden changes in the market value of one of the classes. In this case, the potential to reduce the SCR is even lower as the tendency to invest in Eurozone Bonds for lower returns, shifting afterwards for Peripheral Bonds will be limited. As it is be expected, this is the scenario where the SCR for the different data points is higher, including for the 1,6% return level where the SCR equals 3,12%. Analyzing **Figure 10** , the optimal portfolio will start with a bet in deposits for lower expected returns and, then, in a mix composed by Eurozone and Peripheral Bonds, starting with a greater weight in the first but shifting towards the second as return becomes higher.

Figure 10: **Efficient Frontier - Scenario 3**



For Equity, the pattern is similar as it is used to boost return. However, the 50% cap on investment is expected to influence the portfolio constitution. On the one hand, as

it is the riskiest asset class, the investment limitation will avoid that the SCR increases further as percentage of equities also increases. On the other hand, being the class where rentability is higher, it will not allow that the return increases much further than a particular return.

In short, the optimal asset allocation is quite sensitive to the restrictions imposed to the portfolio, making it hard to reduce the SCR to a great extent. **Table 7** shows overall values obtained for each scenario. However, regardless of the limitations there is a portfolio that, for a given expected return, will result in the lowest SCR possible, which can help the portfolio managers to perceive how they can minimize their capital charges.

Table 7: **Efficient SCR for each Level of Expected Return**

	SCR		
Expected Return	**Scenario 1**	**Scenario 2**	**Scenario 3**
0,50%	3,35%	4,95%	4,04%
1,10%	1,24%	3,50%	3,65%
1,60%	0,02%	2,98%	3,12%
2,10%	0,07%	3,04%	3,14%
2,60%	1,48%	3,91%	4,26%
3,10%	3,68%	5,59%	5,91%
3,60%	5,87%	7,54%	7,75%
4,10%	8,07%	9,59%	9,69%
4,60%	10,27%	11,70%	11,79%
5,10%	12,47%	13,84%	14,89%

Constant bets in Deposits to assure low returns, on Eurozone/Peripheral Bonds for intermediate return levels and on Equities to boost gains will characterize the efficient frontier in its different stages. Corporate Bonds tend to be left out due to the inability to compensate its risk in terms of return; as Eurozone and Peripheral Bonds offer similar returns that can be leveraged with small positions in Equity, it becomes inefficient to maintain the investment in this asset class. Details on asset allocation can be found in **Additional Appendix 9.** Lastly, even though this analysis is only based on the classes of assets held in the portfolio and provides the average capital requirements of all securities included, it provides to the manager the possibility of including securities

that are not in the portfolio, as long as they belong to that asset class. One should, however, be aware that this analysis depends substantially on the assumptions for each one of the asset classes and is made on an average basis, implying that to replicate these results, the manager should be careful in choosing a balanced set of securities as, even within each asset class, capital charges is different among securities. Moreover, the results obtained are dependent on the projections for expected return of each asset class, which, for the period in consideration, are unusually high for safe assets as Eurozone Bonds. This scenario enables the manager to reach greater returns at lower levels of capital requirements.

8 Conclusion

In this paper, we apply the Standard Model developed by EIOPA for the calculation of the risk capital requirements of BPI Vida e Pensões' portfolio, reaching an overall amount of 239,44M€. The results show that risk arising from the asset side accounts for most SCR, mainly due to the Market and Default Risk. Within Market Risk, the main contributors are Spread and Concentration, followed by Interest Rate Risk, arising from the portfolio bias towards fixed income securities and over-exposure to a limited number of entities.

In addition, we decompose the SCR by security, presenting the marginal capital charges per asset which allows for comparisons among assets. Thus, we are able to evaluate the risks driving each asset and know which particular features are key in differentiating securities. The results show that Corporate Bonds and Deposits lead the ranking in terms of capital charges. On the other end, Government Bonds, regardless of the significant portfolio weight, are perceived as being a risk-free class, facing null shocks in most of the risk modules.

Moreover, capital charges are in general lower for short-dated and highly rated debt and Covered Bonds as predicted by Fitch Ratings. Nevertheless, our results show that the expected shift away from short-dated Paper and towards Deposits is not supported by the analysis. Furthermore, Concentration and Default Risk charges may be significantly reduced by decreasing exposure (of Term and Cash Deposits respectively) to the parent bank BPI.

Also, we show that diversification gains are significant for the portfolio (122M€) given that the correlation between risk modules is different from one. By showing that SCR is sensitive to changes in the correlation matrices, the importance of choosing an accurate correlation matrix is reinforced, both to EIOPA or when developing internal models.

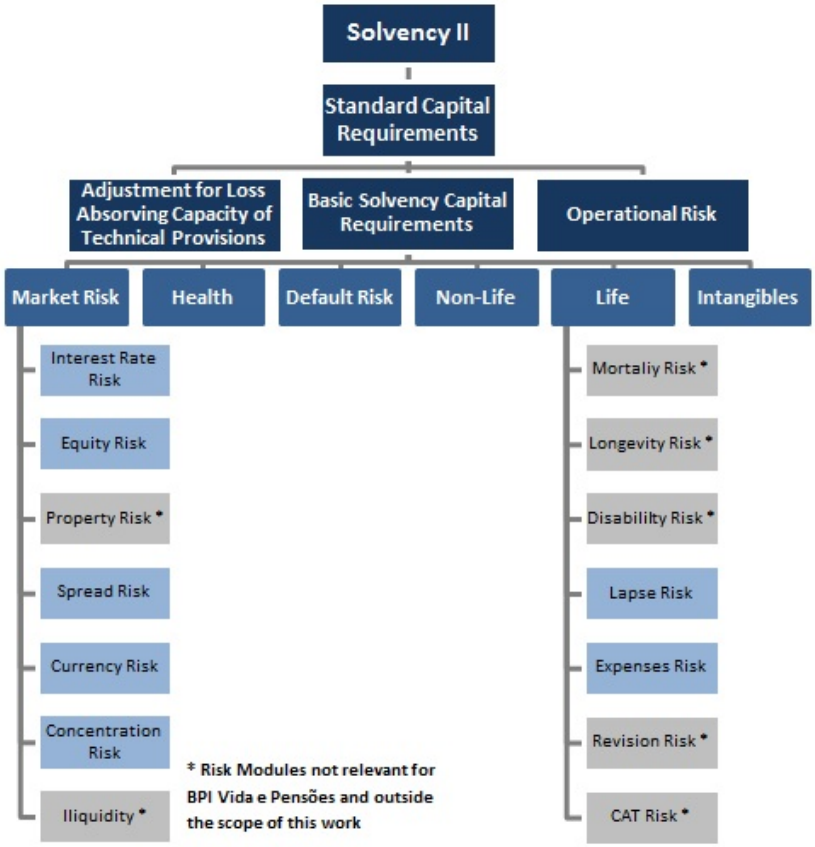
Finally, the risk-return trade-off plays an important role in this analysis as safer assets tend to provide low returns. Therefore, creating a bias towards these safer

classes will put pressure on expected returns. In fact, the main concern of the manager should be the optimization of the portfolio composition given a pre-determined level of return required, where, by doing so, he is able to choose the most capital efficient asset classes. For BPI Vida e Pensões, the computation of the portfolio's efficient frontier shows that an optimal portfolio should: bet on Deposits to assure low expected returns, on Eurozone/Peripheral Bonds for intermediate return levels and, for high returns, the bet should shifts towards Equities, which are more profitable securities but increase as well the level of capital requirements. However, these results depend on the unusual high return of the fixed income securities in the period in analysis.

In summary, this work provides a complete analysis of the risks faced by BPI Vida e Pensões, exploiting the capital requirements up to security level. Moreover, it is also extended to related issues as diversification and optimal allocation. Nevertheless, it is clear that while some of the results are extended to different portfolios, there are particular features not as straightforward and that depend on the portfolio considered, being subject to different results when applying this model to a different set of securities.

9 Appendix

1. Standard Capital Requirement Risk Decomposition



2. Basic Solvency Capital Requirement Correlation Matrix

	Market	**Default**	**Life**	**Operational**
Market	100%			
Default	25%	100%		
Life	25%	25%	100%	
Operational	100%	100%	100%	100%

3. Market Risk Correlation Matrix

	Interest Rate	Equity	Property	Spread	Currency	Concentration
Interest Rate	100%					
Equity	50%	100%				
Property	50%	75%	100%			
Spread	50%	75%	50%	100%		
Currency	25%	25%	25%	25%	100%	
Concentration	0	0	0	0	0	100%

4. Default Risk

4.1. LGD for Derivatives

For a derivative i, the LGD is calculated as:

$$LDG_i = max(0; 90\%(MarketValue_i + RM_{fin,i}) - F \times Collateral_i$$

where MarketValue$_i$= Value of the derivative i; RM$_{fin,i}$ is the risk mitigating effect on market risk of the derivative i; $Collateral_i$ is the Risk-adjusted value of collateral and F is a factor that takes into account the economic effect of the collateral arrangements to the arrangement in case of a credit event of counterparty i.

The Risk Mitigating effect is defined as the difference between the following capital requirements: the capital requirement for market risk when the risk mitigating is not taking into account (an hypothetical SCR, named SCR^{hyp}, and the value of the capital requirement for market risk when the risk mitigating effect is considered. For instance, to calculate the risk mitigating effect of a forward (which will only be affect the value of SCR^{mkt} through currency risk), we consider the hypothetical market risk when the forward is not included (that is, the forward does not mitigate any currency risk) and calculate the difference from the capital requirement for the market risk when the forward is properly included in the computations for currency risk.

4.2. Variance of the loss distribution

The Variance of the loss distribution of Type 1 exposure must equal the sum between $V_{inter}$and V_{intra}, where:

$$V_{inter} = \sum_{(j,k)} \frac{PD_k \times (1 - PD_k) \times PD_j \times (1 - PD_j)}{1.25 \times (PD_k + PD_j) - PD_k \times PD_j} \times TLGD_j \times TLGD_k$$

with (j, k) covering all the possible combinations between companies and $TLGD_j$and $TLGD_k$denoting the total LGD for each single exposure. V_{intra}, at a single name level, is given by:

$$V_{intra} = \sum_{j} \frac{1.5.PD_j \times (1 - PD_j)}{2.5 - PD_j} \times \sum_{PD_j} LGD_i^2$$

5. Marginal

Leblanc (2011)'s methodology is based on the idea that the percentage shock for a security given a specific risk factor is given by the market value of the capital requirement demanded over the total market value invested of the portfolio and that it also corresponds to the weight of $p_j$times the shock applied, that is:

$$\%SCR_j^l = \frac{MVSCR_j^l}{MV} = \frac{MVSCR_j^l}{MV} \times \frac{MV_j}{MV_j} = p_j \times U\%SCR_j^l,$$

where $U\%SCR_j^l$ corresponds to the percentual shock applied.

So, the total percentual value of the SCR, $\%SCR^2$is given by:

$$\%SCR^2 = \frac{MVSCR^2}{MV^2} = \sum_l \sum_m \rho_{lm} \frac{MVSCR^l}{MV} \frac{MVSCR^m}{MV} =$$

$$= \sum_l \sum_m \rho_{lm} \frac{\sum_{j=1}^{N} MVSCR_j^l}{MV} \frac{\sum_{k=1}^{N} MVSCR_k^m}{MV} =$$

$$= \sum_{l}\sum_{m}\rho_{lm}\sum_{j=1}^{N} p_j U\%SCR_j^l \sum_{k=1}^{N} p_k U\%SCR_k^m$$

For further results, it is assumed that $U\%SCR_j^l = SCR_j^l$and $\%SCR = SCR$. Additionally, the results presented are based on the fact that the matrix arising from $\rho_{ml}SCR_j^l SCR_k^m$ is diagonally simmetric when $j \neq k$.

Consequently, and starting off with SCR^2, one has $SCR^2 = \sum_{j=1}^{n} p_j \sum_{k=1}^{n} p_k \sum^{l,m} SCR_l{}^j SCR_m^k \rho_{l,m} = \sum_{k=j} p_j^2 \sum_{l,n} SCR_l^j SCR_j^k p_{l,m} + \sum_{j\neq k}^{n} p_j \sum_{k}^{n} p_k \sum_{l,m} SCR_l^j SCR_m^k \rho_{l,m}$

$$\frac{\partial SCR^2}{\partial p_j} = 2p_j \sum_{l,m} SCR_l^j SCR_m^k \rho_{l,m} + 2\sum_{j\neq k}^{n} p_k \sum_{l,m} SCR_l^j SCR_m^k \rho_{l,m} =$$

$$= 2\sum_{k=1}^{k} p_k \sum_{l,m} SCR_l^j SCR_m^k \rho_{l,m}$$

And $\frac{\partial SCR}{\partial SCR^2} = \frac{1}{2SCR}$, which implies that $\frac{\partial SCR}{\partial SCR^2} \times \frac{\partial SCR^2}{\partial p_j} = \frac{2\sum_{k=1}^{k} p_k \sum_{l,m} SCR_l^j SCR_m^k \rho_{l,m}}{2SCR} = \frac{\sum_{k=1}^{k} p_k \sum_{l,m} SCR_l^j SCR_m^k \rho_{l,m}}{SCR}$, independent of p_j and giving us the derivative of SCR relative to p_j, the investment of j in terms of the overall portfolio.

Then, $\sum_{j=1}^{N} p_j \times \frac{\partial SCR}{\partial p_j} = \sum_{j=1}^{N} p_j \times \frac{\sum_{k=1}^{N} p_k \sum_{l,m} SCR_l^j SCR_m^k \rho_{lm}}{SCR} = \frac{SCR^2}{SCR} = SCR$

6. Expected Return

Asset Class	**Expected Return**
Portuguese Equities	7,10%
Global Emerging Markets Equities	7%
Investment Funds	5,50%
High Yield European Bonds	2,80%
Peripheral Eurozone	2,20%
Investment Grade European Bonds	2%
Eurozone Bonds	1,70%
Deposits	0,07%
Cash	0%

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Additional Appendix

1. Operational Risk

Operational Risk corresponds to the changes in value arising from operational losses, which can be closely related to quality management as, even though they are not taken willingly nor can they be avoided, the firm can attempt to minimize its effects . In the Solvency II framework, these losses may be related to failures in internal processes, people and systems or external occurrences (which include legal risks), other than financial risks arising from expected losses from the insurance company. It is calculated as a percentage of expenses, premiums or mathematical provisions. Unit links are valued at 25% of the expenses, while risk undertaken by the insurer corresponds to the maximum between the operational risk from provisions and premiums. The total risk is limited by a 30% Basic SCR cap.

$$SCR_{Op} = min(30\% \times BSCR; Op) + 0.25 \times Exp_{ul}$$

, being Op determined as $max(Op_{Premiums}; Op_{Provisions})$ where

$Op_{Premiums} = 4\% \times (Earn_{life} - Earn_{life-ul}) + 3\% \times Earn_{non-life} + 4\% \times max(0, Earn_{life} - Earn_{life-ul} - 1.2 \times (pEarn_{life} - pEarn_{life-ul})) + 3\% \times max(0, Earn_{nonlife} - 1.2 \times pEarn_{life})$ and

$$Op_{Provisions} = 0.45\% \times max(0, TP_{life} - TP_{life-ul}) + 3\% \times max(0, TP_{non-life})$$

2. Life Underwriting Risk

Life Underwriting risk is related to the risks associated with the fact that the undertaker (insurance company) provides the individual with life insurance. Therefore, this section includes the capital requirements for several sub-modules such as mortality, longevity, disability/morbidity, lapse, expense, revision and catastrophe. Given the specificities provided by EIOPA, when mortality and longetivity related risks outmatch each other,

they should not be considered, making lapse and expense the only relevant risks to consider. As for Market risk, the total SCR for life underwriting includes the correlations between the different sub-risks and can be expressed as:

$$\sqrt{\sum CorrLife^{rxc} \times Life_r \times Life_c}$$

2.1 Lapse Risk

Lapse risk corresponds to the risk of losses or material changes in the liabilities side caused by the exercise of contractually legal policyholder options such as the termination, surrender, decrease or suspension of the insurance cover or allow insurance policy to lapse.

The capital requirement is, then, given by

$$max(nLapse_{down}; nLapse_{up}; nLapse_{mass})$$

For the calculation of both $nLapse_{down}$and $nLapse_{up}$it is considered a 50% permanent and immediate shock over the exercise rates of all relevant options in all future years.However, in the case of the downward shock, the decrease in rates cannot exceed 20 percentage points, while in the upward shock, the resulting increased exercise rates cannot exceed 100%. As for $nLapse_{mass}$it specifies a combination between a discontinuation of 70% of the insurance policies where such discontinuation would result in an increase of technical provisions and the policyholders would either be not natural persons where the discontinuation of the policy is not subjected to approval or a natural person acting for the benefit of the beneficiaries under these policies and 40% over other insurance policies. In this context, discontinuance means surrender, lapse without value or exercising other discontinuity options (or not applying continuity ones).

2.2 Expense Risk

Expense Risk arises from changes in the level of expenses incurred associated to servicing insurance contracts, corresponding to the variation in the BSCR given a specific

shock. This shock is obtained from the combination between two permanent changes an increase of 10% in the amount of expenses and an 1 percentage point increase to the expense inflation rate. Consequently,

$$Life_{exp} = \Delta BOF|expshock$$

3. Property Risk

Property risk measures the sensitivity of assets, liabilities and financial investments to the level of volatility of property market values, considered to be land, buildings and immovable property right or property investment carried by the insurance company. Any other real estate investment that is not included in any of these classifications should be treated as an equity investment. As it occurs for other risks computations, the capital requirements for property risk correspond to the maximum value between the change in Net Asset Value after the application of an instantaneously negative shock of 25% in the value of the investments and 0.

4. Smith-Wilson Method

The Smith Wilson method is a macroeconomic model where the yield curve is fitted to observed prices of financial instruments, having as input long term macroeconomic expectations. It is usually used to extrapolate adjusted spot rates for future dates from the zero coupon rates already present in the market. Its pricing function P(t), for all t>0, is set as the sum of $e^{-UFR.t}$ to account for the asymptotical long term behavior of the discount factor and a linear combination of N kernel functions. These kernel functions are appropriately defined functions of the input market data and two other input parameters: the ultimate forward rate (UFR) and a parameter *alpha* that determined the convergence towards the UFR.

This method is already used by EIOPA to build the term structure supplied to the insurance portfolio managers and that is the main reason why it was chosen for

the extrapolation of interest rates for longer maturities in this thesis. Alongside with the linear interpolation method used for lower maturities and between data points, it allows for the construction of a interest rate term structure further adjusted for credit risk. This was a required step as EIOPA only makes this curves available to manager for periods where reporting is required. Consequently, by computing the adequate rates and also performing the required adjustment, this thesis offers the possibility of managers evaluating their portfolios at any time using the most relevant term structure.

5. Upward and Downward shocks for Interest Rate Risk

The new term structures are obtained by multiplying the current curve by $(1 + s^{up})$and $(1 + s^{down})$. However, the absolute increase in the rates in the upward scenario shall be of, at least, one percentage point.

Maturity t (in years)	**Relative Change** $s^{up}(t)$	**Relative Change** $s^{down}(t)$
1 or Shorter	70%	-75%
2	70%	-65%
3	64%	-56%
4	59%	-50%
5	55%	-46%
6	52%	-42%
7	49%	-39%
8	47%	-36%
9	44%	-33%
10	42%	-31%
11	39%	-30%
12	37%	-29%
13	35%	-28%
14	34%	-27%
15	33%	-28%
16	31%	-28%
17	30%	-28%
18	29%	-28%
19	27%	-29%
20	26%	-29%
90 or Longer	20%	-20%

6. Spread Risk: CQS and Risk Factors

In order to know the factor that should be applied to each fixed income security, EIOPA provides a set of tables which depend on both the duration of the security (vertically presented) and its CQS (horizontally in the tables below). For the Variable Spread Risk Factors, each factor should be multiplied by its duration minus the lower bound of its class, i.e. a bond with a duration lower than 5 has its variable shock multiplied by its duration, while a bond with a maturity higher than 5 years but lower than 10 will face a shock of x% times duration - 5 and so on.

Variable Spread Risk Factors For Bonds (x Duration)							
	0	**1**	**2**	**3**	**4**	**5**	**6**
5	0,9%	1,1%	1,4%	2,5%	4,5%	7,5%	7,5%
10	0,5%	0,6%	0,7%	1,5%	2,5%	4,2%	4,2%
15	0,5%	0,5%	0,5%	1,0%	1,8%	0,5%	0,5%
20	0,5%	0,5%	0,5%	1,0%	0,5%	0,5%	0,5%
25	0,5%	0,5%	0,5%	0,5%	0,5%	0,5%	0,5%

Fixed Spread Risk Factors							
	0	**1**	**2**	**3**	**4**	**5**	**6**
5	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
10	4,5%	5,5%	7,0%	12,5%	22,5%	37,5%	37,5%
15	7,2%	8,4%	10,5%	20,0%	35,0%	58,5%	58,5%
20	9,7%	10,9%	13,0%	25,0%	44,0%	61,0%	61,0%
25	12,2%	13,4%	15,5%	30,0%	46,6%	63,5%	63,5%

Variable Risk Factors for Unrated Bonds (x Duration)	
5	3,0%
10	1,7%
More than 20	1,2%

Fixed Risk Factors for Unrated Bonds	
5	0,0%
10	15,0%
More than 20	23,5%

Type 1 Securitization Risk Factors			
0	**1**	**2**	**3**
2,10% x Dur.	4,20% x Dur.	7,40% x Dur.	8,50% x Dur.

Type 2 Securitization Risk Factors						
0	**1**	**2**	**3**	**4**	**5**	**6**
12,50%	13,40%	16,60%	19,70%	82%	100%	100%

Risk Factor for Covered Bonds		
	0	**1**
Up to 5 Years	0,7% x Duration	0,9% x Duration
More Than 5 Years	Min(3,5%+0,5%(Duration -5),1)	Min(4,50%+0,5%(Duration-5),1)

7. Credit Quality Step Table

Agency Rating	Credit Quality Step
AAA	0
AA+, AA, AA-	1
A+, A, A-	2
BBB+, BBB, BBB-	3
BB+, BB, BB-	4
B+, B, B-	5
CCC+, CCC, CCC-, C, D	6
ND	Unrated

8. Concentration Risk

Credit Quality Step	Relative Excess Exposure Threshold (CT)	Risk Factor (g)
0	3%	12%
1	3%	12%
2	3%	12%
3	1,5%	27%
4	1,5%	73%
5	1,5%	73%
6 or Unrated	1,5%	73%

9. Allocation Bets

Scenario 1									
SCR	**Portuguese Equities**	**Global Emerging Markets Equities**	**Eurozone Bonds**	**Peripherical Eurozone**	**High Yield European Bonds**	**Investment Grade European Bonds**	**Cash**	**Investment Funds**	**Deposits**
3,35%	0,00%	0,00%	26,47%	0,00%	0,00%	0,00%	0,00%	0,00%	74,50%
1,24%	0,00%	0,00%	63,22%	0,00%	0,00%	0,00%	0,00%	0,00%	37,75%
0,02%	0,44%	0,60%	89,31%	0,00%	0,00%	0,02%	0,00%	0,01%	10,59%
0,00%	0,03%	1,83%	96,77%	0,00%	0,00%	0,00%	0,00%	2,34%	0,00%
0,07%	0,15%	2,26%	47,46%	51,10%	0,00%	0,00%	0,00%	0,01%	0,00%
1,48%	0,00%	7,89%	0,00%	93,08%	0,00%	0,00%	0,00%	0,00%	0,00%
3,68%	0,00%	18,31%	0,00%	82,66%	0,00%	0,00%	0,00%	0,00%	0,00%
5,87%	0,00%	28,72%	0,00%	72,25%	0,00%	0,00%	0,00%	0,00%	0,00%
8,07%	0,00%	39,14%	0,00%	61,83%	0,00%	0,00%	0,00%	0,00%	0,00%
10,27%	0,00%	49,56%	0,00%	51,41%	0,00%	0,00%	0,00%	0,00%	0,00%
12,47%	0,00%	59,97%	0,00%	41,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Scenario 2									
SCR	**Portuguese Equities**	**Global Emerging Markets Equities**	**Eurozone Bonds**	**Peripherical Eurozone**	**High Yield European Bonds**	**Investment Grade European Bonds**	**Cash**	**Investment Funds**	**Deposits**
4,95%	0,00%	0,00%	26,60%	0,00%	0,00%	0,00%	3,01%	0,00%	71,36%
3,50%	0,00%	0,00%	63,34%	0,00%	0,00%	0,00%	3,01%	0,00%	34,62%
2,98%	0,00%	0,00%	93,96%	0,00%	0,00%	0,00%	3,01%	0,00%	4,00%
2,97%	0,16%	0,37%	97,42%	0,00%	0,00%	0,00%	3,01%	0,01%	0,00%
3,04%	0,00%	0,00%	11,03%	86,93%	0,00%	0,00%	3,01%	0,00%	0,00%
3,91%	0,00%	9,27%	0,00%	88,69%	0,00%	0,00%	3,01%	0,00%	0,00%
5,59%	0,00%	19,68%	0,00%	78,27%	0,00%	0,00%	3,01%	0,00%	0,00%
7,54%	0,00%	30,10%	0,00%	67,86%	0,00%	0,00%	3,01%	0,00%	0,00%
9,59%	0,00%	40,52%	0,00%	57,44%	0,00%	0,00%	3,01%	0,00%	0,00%
11,70%	0,00%	50,93%	0,00%	47,02%	0,00%	0,00%	3,01%	0,00%	0,00%
13,84%	0,00%	61,35%	0,00%	36,61%	0,00%	0,00%	3,01%	0,00%	0,00%

Scenario 3									
SCR	**Portuguese Equities**	**Global Emerging Markets Equities**	**Eurozone Bonds**	**Peripherical Eurozone**	**High Yield European Bonds**	**Investment Grade European Bonds**	**Cash**	**Investment Funds**	**Deposits**
4,04%	0,00%	0,00%	47,96%	0,00%	0,00%	0,00%	3,01%	0,00%	50,00%
3,65%	0,00%	0,00%	50,00%	10,21%	0,00%	0,00%	3,01%	0,00%	37,75%
3,12%	0,00%	0,00%	50,00%	33,65%	0,00%	0,00%	3,01%	0,00%	14,31%
2,99%	0,00%	0,00%	50,00%	47,96%	0,00%	0,00%	3,01%	0,00%	0,00%
3,14%	0,00%	3,48%	44,48%	50,00%	0,00%	0,00%	3,01%	0,00%	0,00%
4,26%	0,00%	12,92%	35,04%	50,00%	0,00%	0,00%	3,01%	0,00%	0,00%
5,91%	0,00%	22,35%	25,61%	50,00%	0,00%	0,00%	3,01%	0,00%	0,00%
7,75%	0,00%	31,79%	16,17%	50,00%	0,00%	0,00%	3,01%	0,00%	0,00%
9,69%	0,00%	41,22%	6,74%	50,00%	0,00%	0,00%	3,01%	0,00%	0,00%
11,79%	0,00%	50,00%	0,00%	46,60%	0,00%	0,00%	3,01%	1,36%	0,00%
14,89%	0,00%	50,00%	0,00%	31,44%	0,00%	0,00%	3,01%	16,52%	0,00%